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New look at studying simple harmonic oscillations with a unified approach

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Abstract: This work describes the methodology of studying simple harmonic oscillations with a unified approach. It is shown that, in case of each oscillatory system of a different origin the following parameters are set: the form of motion performed, extensive and intensive quantities that characterize this form of motion; subsystems of capacitive, resistive and inductive nature and relations that characterize them; differential equations on the basis of relevant extensive quantities and their derivatives. As a result, it appears that the analogue of the m mass in a spring pendulum is not the L inductance in an electric oscillating circuit, but the C capacity; the analogue of inductance is the elasticity coefficient 1/k.

Key words: *Subsystems of capacitive; Resistive and inductive nature; Momentum capacity; Momentum current; Mass current; Inductance of the system*

1. Introduction

The identification of possibilities and ways to apply a unified approach in the study of simple oscillations, included in the physics program of technical universities, is of great importance for teaching of this subject.

Speaking of the unified approach we mean definition of identical physical properties and relations, subsystems performing the same function in oscillations of various origins and creation of physical and mathematical models on the basis of these analogs. In our opinion, this approach to teaching students results in giving them quality knowledge.

For the implementation of a unified approach in the considered oscillatory systems let us allocate appropriate extensive and intensive properties (potentials) related to them (Çengel and Boles, 2006) Oscillatory systems perform certain forms of motion and their energy is determined by an extensive quantity. Therefore, the extensive quantity in the appropriate form of motion is also called an energy carrier. For example, during the rotation of a body around a fixed axis an angular momentum is the energy carrier, etc. (Etkin, 2008). Table 1 shows some characteristics of oscillatory processes that are commonly used in equipment and are included in the physics course of technical universities.

As can be seen, each of these oscillatory systems is characterized by only one form of motion. In this sense, they are simple harmonic motions (SHM).

It is known that the process of oscillation is a repetition of changes in properties (physical quantities) of a system depending on the time. In a physical system consisting of two subsystems of capacitive nature and connecting its subsystems of resistive nature (RC system), in case of a difference in potentials of these subsystems, there is a flow of energy carrier in the direction from the large potential to the small potential. In an RC system, if the potentials of the capacitive subsystems are equal, the flow of energy carrier is stopped and does not repeat (Safarov and Valiev, 2013). Therefore, in order for oscillations to occur in a physical system, aside from subsystems of resistive and capacitive nature, we also need a subsystem of an inductive character. The following conditions must be met for the occurrence of oscillations in a physical system:

- the oscillatory system should include two subsystems (of capacitive nature), where the physical property (quantity), able to flow back and forth, can be accumulated;
- the oscillatory system must include a subsystem of an inductive nature, in order to create a cause for counteracting with the system, in order for it to remain in the existing state; as a result, the energy carrier flow (an extensive quantity) (Herrmann and Bruno Schmid, 1985) with equal potentials of capacitive subsystems, does not stop and keeps going;
- in an oscillatory system resistance to the back and forth flow of physical quantities must be small.

2. Subsystems of oscillatory systems

To meet all these conditions, each of the simple oscillatory systems should consist of three subsystems: I) that of the capacitive nature, II) that of the resistive nature; III) that of the inductive

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nature. A block diagram of a simple oscillatory system can be represented as follows:

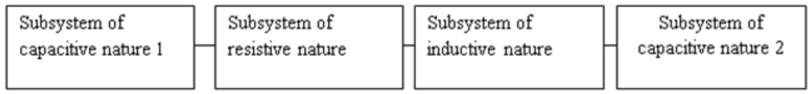

Fig.1 : System block diagram

Table 1: characteristics of oscillatory processes that are commonly used in equipment

Oscillation process	Form of motion	Extensive quantity	Intensive quantity
a) oscillation of a rigid body (spring pendulum) suspended on a spring attached to the support	linear motion (translation)	linear momentum	linear velocity
b) oscillation of the piston in a vertically disposed cylindrical vessel filled with gas	linear motion (translation)	linear momentum	linear velocity
c) oscillations of a solid body of a specific shape (physical pendulum) suspended on a point that does not coincide with the center of gravity	rotation	angular momentum	angular speed
d) oscillation of the disk on a horizontal plane, suspended on a thread (torsion pendulum) that's elastic to the torsion	rotation	angular momentum	angular speed
e) oscillation of the liquid column in the vessels, the bottom of which is connected to the valve with a tube (U-shaped tube)	laminar motion	liquid volume	pressure
g) oscillation of electrical charges in a circuit containing a capacitor and a coil (electric oscillating circuit)	electricity	electric charge	electric potential

Now let us define the subsystems for different oscillatory systems.

In an oscillatory system, the subsystems of capacitive nature provide the accumulation of energy of a corresponding form of motion and they are as follows: a) in a spring pendulum a body suspended on a spring and the spring attachment bearing on Earth (mass – the momentum capacity is infinitely large) (fig. 1a); b) during the piston oscillation the bearing of cylindrical vessel on Earth and the piston of a certain mass (fig. 1b); c) body and the Earth in a physical pendulum (fig. 1c); d) in the torsion pendulum a disk oscillating on a horizontal plane and the Earth (fig. 1d); e) left and right vessels during oscillation of the liquid column (fig. 1e); f) capacitor plates in the electric oscillating circuit (fig. 1f).

In an oscillatory system, resistive subsystems ensure energy carrier flow of an appropriate form of back and forth motion between the two subsystems of capacitive nature. The cause of the flow is the difference of potentials between the subsystems of capacitive nature. Subsystems of resistive nature are the following: a) in a spring pendulum the spring and its attachment bearing create resistance to the flow of linear momentum determined by the law, similar to Ohm's law (Herrmann and Bruno Schmid, 1985, Plappert D, 1972) G b) during piston oscillation the air column under the piston and bearing of the cylindrical vessel create resistance to the flow of linear momentum determined by the law, similar to Ohm's law, c) in a physical pendulum the body itself and the bearing create resistance to the flow of angular momentum determined by the law, similar to Ohm's law, d) in the torsion pendulum a torsion elastic thread and bearing create resistance to the flow of angular momentum determined by the law, similar to Ohm's law, e) during the oscillation of liquid column the tube connecting the vessels, creates resistance to the volume (quantity) flow of liquid determined by the law of Poiseuille (Hans U. Fuchs, 2010) f) in an electric oscillating circuit wires and winding connecting the capacitor plates, create resistance to electric current determined by Ohm's law.

In an oscillatory system, the subsystem of inductive nature creates a potential difference proportional to the speed of change of energy carrier flow and equal in magnitude of the capacitive potential difference (negative), to the initial moment of oscillation. Subsystems of inductive nature are the following: a) in a spring pendulum the spring (as well as the spring material) create resistance to the momentum flow, b) during the oscillation of the piston the air column under the piston; c) a body suspended on a physical pendulum; d) in the torsion pendulum a thread elastic to torsion; e) during the oscillation of liquid column the liquid in the tube connecting vessels, f) the conductor coil in the electric oscillating circuit.

Table 2 shows the capacitive and resistive characteristics of subsystems of the considered oscillatory systems.

Table 2: the capacitive and resistive characteristics of subsystems of the considered oscillatory systems

Currents of extensive quantities	Capacity of extensive quantities (properties)	Capacitive potential difference	Resistance to the current of extensive quantities (properties)	Resistive potential difference
a) and b) linear momentum current (force)	body mass [8]	$v_C = \frac{p}{m}$ (1*)	R	$v_R = R \cdot F$ (2*) This ratio takes place in the work of a damper
c) and d) angular momentum current (M - torque)	moment of inertia J	$\omega_C = \frac{L}{J}$ (1**)	R	$\omega_R = R \cdot M$ (2**)
e) flux of volume I_v f) electrical current I	$V/P = K_v$ [1, p.34] Electrical capacity C	$P_C = \frac{V}{K_v}$ (1***) $U_c = \frac{Q}{C}$ (1****)	$R = \frac{8\eta l}{\pi r^4}$ (6) Poiseuille's law $R = \rho \frac{l}{S}$	$P_R = R \cdot I_v$ (2***) $U_R = R \cdot I$ (2****)

In Tables 1 and 2, the contents of each column are physical analogs. A unified approach can be implemented on the basis of these analogs.

It should be kept in mind that each of the oscillation processes in question occurs in a closed circuit of the corresponding flow. During oscillation electric current runs through and the elements of the subsystem create a difference of potentials (drop of voltage), resisting the flow of current.

3. Physical and mathematical description of the subsystems of the oscillatory system

And now let us try to give physical and mathematical description to the subsystems of an inductive nature. Let us look at different oscillatory systems:

a) in a spring pendulum to start the oscillation process compress the spring and remove the body suspended onto its end from the state of equilibrium and release the spring while releasing the body. Upon compression of the spring work is done against the spring's elastic force and the spring gains potential energy. By converting the potential energy of the compressed spring into the body's kinetic energy the potential difference (velocity difference) is applied between the body and the Earth. The moment of full conversion of the potential energy of a compressed spring into the kinetic energy of the body corresponds to the time t = 0. At this point, the body (subsystem 1) has a maximum speed relative to the bearing – Earth. This potential difference is of capacitive nature and shall be denoted via V_C. Starting from time t = 0, the momentum current starts running through the spring (the springs starts to stretch thus causing the emergence of elastic force). The speed difference V_C can be associated with several effects. V_C can be reduced in absolute value by V_R – the resistive velocity difference, resulting from the resistance to momentum current.

Capacitive velocity difference V_C also may increase or decrease by V_L - inductive velocity difference, arising from changes in momentum current with time. In this case, two extreme cases are possible: I) the current does not change and the capacitive velocity difference, created in the beginning, is only connected to resistance, i.e, $V_C = -V_R$; II) the current is zero and the capacitive velocity difference is only connected to the rate of current change, i.e., $V_C = -V_L$;

In a spring pendulum elastic force emerges and is bound by the elasticity of spring (momentum current) and can be calculated using Hooke's law. Performing some transformations in the mathematical expression of the law, it is possible to obtain a formula of velocity difference (potential difference), responsible for the change in the elastic force depending on time. To do this, perform the following transformations:

$$F = -kx\ ; \qquad \dot{F} = k\dot{x}; \qquad \ddot{p} = -k v_L; \qquad v_L = -\frac{1}{k}\ddot{p} \qquad (3^*)$$

As can be seen, the obtained velocity difference is proportional to the change rate of momentum current. In the electric current such ratio is bound by the inductive nature of the current. As an analog, the momentum current also has an inductive nature.

In general, the capacitive velocity difference, created at the beginning, is bound by two events at the same time, in other words V_C is divided between the two events, i. e.

$$V_C = -V_R - V_L \qquad (4^*)$$

Keep in mind that V_R – resistive velocity difference, can only be negative in the direction of flow, and V_L – inductive velocity difference, can be negative or positive. A positive value V_L means that the momentum current decreases depending on time.

For the momentum current a spring plays a role of a resistor and an inductive element. The velocity difference (capacitive potential difference) applied to the body at the time t=0, by being completely converted into inductive velocity difference, changes the momentum current (force). Therefore, initially the inductive velocity difference is maximum, the resistive velocity difference amounts to zero and according to ratios (2), we get $F(0) = \dot{p}(0) = 0$. When moving away from the initial moment, the momentum current running through a spring from the subsystem body to the subsystem Earth increases, and so does V_R respectively to ratios (2). At the time t=T/4, V_L=0, V_C=-V_R, i.e. the momentum current (force) becomes maximum, and its change equals zero. The influence of the spring inductance property is reduced to zero.

As seen, from time t=0 to t=T/4, the momentum current running through the spring into Earth (the force onto the bearing – Earth system) does not increase from zero immediately, but gradually. In other words, in order to achieve the value of momentum current from zero to the maximum a finite amount of time is required.

Since the moment t= T/4 the momentum current starts to decrease. The current starts changing and V_L and V_R are different from zero. At the moment t=T/2, V_L=0, the capacitive potential difference is -V_C, the resistive velocity difference is V_R=V_C. At this point, the momentum applied to the body through external energy, is fully returned to the subsystem Earth.

The second half-period starts with the flow of momentum current from the Earth to the body (the body begins to acquire a positive momentum), i.e., the process begins and is repeated in the opposite direction. At the moment of time 3T/4, the spring reaches its maximum compression, the potential energy of the spring becomes maximum. At the moment of time T, this energy is completely converted into the kinetic energy of the body and it obtains the maximum momentum (mV_C).

It is clear from the above that the capacitive potential difference (V_C), transmitted to the subsystem of capacitive nature initially from the external energy sources is used for the resistive (V_R) and inductive (V_L) potential differences. In fact, we are talking about a closed circuit of momentum current. According to Kirchhoff's law derived from the law of conservation and transformation of energy the following condition is met in a closed circuit:

$$V_C + V_R + V_L = 0$$
(5 *)

b) to obtain piston oscillations it is necessary to compress the gas by moving the piston's center of gravity down and release the piston. In this case, the compressed gas itself acts as an elastic spring. The compression of gas also produces force proportional to the displacement (x) of the piston's center of gravity. Making the corresponding transformations in the mathematical expression of this force, we can obtain the formula of velocity difference (potential difference), responsible for the change in force depending on time, which is an analog of formula (3*). Here, however, k is replaced by a coefficient, determined by parameters of gas (Ricardsons metod) (Stephen et al, 2010).

$$k = \left(\frac{\gamma \cdot P \cdot S^2}{V} \right)$$
(7)

γ – ratio of gas heat capacity, P – gas pressure and V gas volume, S – the piston's cross-section area. The law of conservation and transformation of energy in a closed circuit of momentum current also remains in the form of (5*).

c) for obtaining oscillations of a body in a physical pendulum, it should be moved to the right from its equilibrium position and released. The potential energy of the body will then be transformed into its rotational kinetic energy. Due to the rotational motion of the body an angular momentum emerges as an energy carrier. Continue the arguments as in a) and b). At the moment when t=0, angular velocity difference ω_C is at its maximum value and has a capacitive nature, while the angular momentum current equals zero. In later moments the current gradually increases. The impression is made that the body is a subsystem, which accumulates angular momentum. The angular velocity difference ω_C, applied to the rotating body, may be connected to several effects. This value can be changed by the amount of ω_R – the resistive angular velocity difference arising due to the current's resistance to the angular momentum. The velocity difference may increase or decrease by the amount of ω_L – the inductive angular velocity difference arising due to changes in the angular momentum current depending on time. Two extreme cases are possible here: I) the current does not change and the capacitive velocity difference initially is caused only by the power of resistance, i.e., ω_C = - ω_R; II) the current amounts to zero and the capacitive velocity difference is due to only the rate of current change, i.e., ω_C = - ω_L;

In a physical pendulum arises a moment of force proportional to the body's angle of inclination (θ) from the equilibrium position. Making the corresponding transformations in the mathematical expression of this force, we can obtain the formula of the difference of angular velocities (potential difference) responsible for the change in the moment of force depending on time, which is an analog of formula (3*).

$$M = -mgb \cdot \theta \;; \quad \dot{M} = -mgb \cdot \dot{\theta}; \quad \ddot{L} = -mgb \cdot \omega_L; \quad \omega_L = -\frac{1}{mgb}\ddot{L} \qquad (3^{**})$$

It can be seen that the angular velocity difference is proportional to the rate of change of current angular momentum (torque). In the electric current this ratio is bound by the inductive nature of the current. By analogy, we can come to a conclusion about the inductive nature of the angular momentum current.

In general, the capacitive angular velocity difference, created at the beginning, is simultaneously bound by both events, in other words, ω_C is divided between the two events, i.e.

$$\omega_C = -\omega_R - \omega_L \qquad (4^{**})$$

It should be taken into account that ω_R – the angular velocity difference, arising due to the resistive nature, can only be negative in the direction of flow, while ω_L – the angular velocity difference, arising due to the inductive nature, can be negative or positive. ω_L is positive, which means that the angular momentum current decreases with time.

At the time t = 0 an extreme condition II) occurs while at T/4 an extreme condition I) occurs. The following arguments can be continued like in case of a spring pendulum.

In this case we are talking about a closed current circuit of angular momentum and according to Kirchhoff's law derived from the law of conservation and transformation of energy in a closed circuit, the following condition is met, i.e.

$$\omega_C + \omega_R + \omega_L = 0 \qquad (5^{**})$$

d) in the torsion pendulum a form of motion is similar to that of a physical pendulum, therefore it is possible to argue similarly. In order to achieve oscillations it is necessary to displace the disk from the state of equilibrium at an angle θ, and let it go. At the time t = 0, the disk passes through the state of equilibrium and has a maximum angular velocity ω_C. All ratios for the time 0, t/4, T/2, 3T/4, T are identical to those of the point c).

In the torsion pendulum the moment of force occurs proportionally to the angle of torsion (θ) of the body from the state of equilibrium. Making the corresponding transformations in the mathematical expression of this force, we can obtain the formula of angular velocity difference (potential difference) responsible for the change of the moment of force through time, which is analogous to the formula (3).

$$M = -c\theta\ ; \quad \dot{M} = -c\dot{\theta}\ ; \quad \ddot{L} = -c\omega_L; \quad \omega_L = -\frac{1}{c}\ddot{L} \qquad (3^{***})$$

Here, M is a moment of elastic force, which occurs at torsioning of an elastic thread, c-coefficient, which characterizes the elastic properties of the thread. The ratio between (4**) and (5**) remains the same.

e) in order to study the oscillations of liquid in the connected vessels, it is necessary to understand the nature of pressure difference existing in the various cross sections of a closed tube with the liquid. Imagine that pressure difference arises in the sections connecting the first and second vessels with the tube. This pressure difference may be related to a number of events. The pressure difference may change in the direction of flow and as a result of friction of the liquid on the tube walls. Resulting from the rate of change of the flux of volume, the pressure difference may increase or decrease in the direction from the first to the second section. Two extreme cases are possible in this situation: I) the flux of volume does not change and the capacitive pressure difference arising initially is caused only by the friction force of the liquid, i.e., $P_C = -P_R$; II) the flux of volume is equal to zero and the capacitive pressure difference is caused only by the rate of change of the flux of volume, i.e. $P_C = -P_L$;

In general, the capacitive pressure difference, created at the beginning, is associated with both events at the same time, in other words P_C is divided between two events, i.e.

$$P_C = -P_R - P_L \qquad (4^{***})$$

This condition is a physical analog of the electric circuit where inductors and resistors are separate elements placed one after another i.e., in series, and can be visualized more easily.

It is necessary to take into account that P_R - pressure difference arising due to the resistive nature can only be negative in the direction of flow, and P_L - pressure difference resulting from the inductive nature can be both positive and negative. A positive value of P_L means that the fluid flux is decreasing with time. In the literature [5, p.43] P_L is defined as

$$P_L = -L \cdot \dot{I}_v = -L\ddot{V} \qquad (3^{****})$$

In fact, suppose that the left vessel has the liquid of a certain level, and the vessel on the right is empty. At the time t=0 open the valve in the tube connecting the vessels and let the liquid flow. The right vessel is empty, that is why the liquid begins to flow gradually in the direction from the left to the right vessel and the flux of volume begins to increase from zero not instantly but gradually. The flux of volume achieves a maximum from zero in a finite amount of time. The experiment shows that the time, when liquid levels in the vessels (with equal hydrostatic pressures of liquid columns in the sections) are equilibrated, the flow of liquid doesn't stop, on the contrary, its flux of volume has the largest value. At this time (T/4), the above-mentioned extreme condition I) occurs (P_L=0). The flow of liquid continues (the system doesn't go out of the existing state) due to the inductive properties. Since the flow of liquid doesn't stop, the liquid level increases in the right vessel and continues to decrease in the left vessel. At some point of time (T/2) the level of liquid in the right vessel reaches maximum value and the flow of liquid stops. After this the liquid flows in the opposite direction until the liquid in the left vessel achieves its maximum level. This way the liquid oscillates moving back and forth between the connected vessels.

It is clear from the above that the hydrostatic pressure (P_C) of a column of a certain liquid volume initially filled within the left vessel (subsystem 1 of capacitive nature) due to external sources of energy is used for resistive (P_R) and inductive pressure differences (P_L). In fact, we are talking about a closed circuit of the liquid flow. According to Kirchhoff's law derived from the law of conservation and transformation of energy in a closed circuit, the following condition is met, i.e.,

$$P_C + P_R + P_L = 0 \qquad (5^{***})$$

f) in order to start the electrical oscillations in the oscillating circuit it is necessary to charge the capacitor and concatenate it to the resistor and the coil. As a result of charging a potential difference U_C arises between the plates of the capacitor. According to the block diagram of the oscillation system, the capacitor plates can be considered subsystems of capacitive nature. In fact, one capacitor performs the function of two subsystems of capacitive nature. In order to understand the mechanism, the circuit can be connected to two capacitors. Except the connected vessels, in all other analyzed oscillatory systems, it is not clear that there are two subsystems of capacitive nature. As noted above, in these oscillatory systems the Earth is considered to be the second subsystem.

Experiments show that at t=0, the electric current in the circuit is equal to zero. At this moment the conditions U_C=-U_L and U_R=0 should be met. According to the law of electromagnetic induction U_L is calculated by the formula

$$U_L = -L_e\dot{I} = -L_e\ddot{q} \qquad (3^{*****})$$

At the time T/4 the condition $U_C = -U_R$ and U_L=0 are met. The resistive potential difference is maximum, that's why the electric current is also maximum.

At this moment the capacitive voltage completely converts into the resistive voltage and the electric field energy entirely transforms into magnetic field energy. In all other analyzed oscillations such splitting of motion forms is absent. This is the major difference between electrical and other oscillations. This difference has no influence on a common approach. It's the opposite, two oscillating systems are studied at the same time, in which the electric and magnetic forms of motion take place under the same laws. At the time T/2 the ratios corresponding to the time t=0 are formed and the energy is fully accumulated in the capacitor. In the next moments of time these events are repeated. According to Kirchhoff's law derived from the law of conservation and transformation of energy in a closed circuit, the following condition is met, i.e.,

$$U_C + U_R + U_L = 0 \qquad (5^{****})$$

In oscillatory systems of different nature there are elements of the same physical properties (capacitive, inductive and resistive properties) and taking into account the oscillations of the values, characterizing the same physical properties (extensive property or an energy carrier), it is possible to apply a unified approach to all oscillatory processes of a similar type.

Table 3 shows differential equations of oscillations for the analyzed oscillatory systems. Traditionally, in oscillations with a linear form of motion (for example, in a spring pendulum), the differential equation is written in accordance with the spatial coordinate and its derivatives (Pippard, 2006; Walter Fox Smith, 2010)In table 3, in point *a*) the equation is written in accordance with the linear momentum and its derivatives.

As an example let us analyze the laws of resistive and inductive potential difference and the momentum current in the spring pendulum.

$p(t) = p_C\cos\omega_0 t$; $F(t) = \dot{p} = p_C\omega_0\cos(\omega_0 t + \frac{\pi}{2})$;
$v_R(t) = R\dot{p} = Rp_C\omega_0\cos(\omega_0 t + \frac{\pi}{2})$

$v_L(t) = \frac{1}{k}\ddot{p} = p_C\frac{1}{k}\omega_0^2\cos(\omega_0 t + \pi)$; $v_L(0) = -p_C\frac{1}{k}\omega_0^2 = -p_C\frac{1}{k}\frac{k}{m} = -\frac{p_C}{m} = -v_C$

Table 3: differential equations of oscillations for the analyzed oscillatory systems

a)	$v_C+v_R+v_L=0$; $\frac{p}{m} + R\dot{p} + \frac{1}{k}\ddot{p} = 0$; $\ddot{p} + kR\dot{p} + \frac{k}{m}p = 0$; $\beta = \frac{kR}{2}$; $\omega_0^2 = \frac{k}{m}$, $\ddot{p} + 2\beta\dot{p} + \omega_0^2 p = 0$. $[\beta] = \frac{N}{m}\frac{\frac{m}{s}}{N} = \frac{1}{s}$
b)	equations and transformations are the same as in a). $k = \frac{\gamma P S^2}{V}$; $\omega_0^2 = \frac{k}{m} = \frac{\gamma P S^2}{mV}$; $\beta = \frac{kR}{2} = \frac{\gamma P S^2 R}{2V}$. $[k] = \frac{m^4\frac{N}{m^2}}{m^3} = \frac{N}{m}$; $[\beta] = [k][R] = \frac{N}{m}\frac{\frac{m}{s}}{N} = \frac{1}{s}$.
c)	$\omega_c + \omega_R + \omega_L = 0$; $\frac{L}{J} + R\dot{L} + \frac{1}{k}\ddot{L} = 0$; $\ddot{L} + kR\dot{L} + \frac{k}{J}L = 0$; $k = mgb$; $\omega_0^2 = \frac{k}{J} = \frac{mgb}{J}$. $\ddot{L} + 2\beta\dot{L} + \frac{k}{J}L = 0$. $[\omega_0] = \sqrt{\frac{kg\frac{m}{s^2}m}{kg\,m^2}} = \frac{1}{s}$; $[R] = \frac{\frac{1}{s}}{Nm}$, $[\beta] = kg\frac{m}{s^2}m\frac{1}{Nms} = \frac{1}{s}$. $[k] = kg\frac{m}{s^2}m$
d)	equations and transformations are the same as in c). Instead of the *k* coefficient the *c* coefficient is applied: $\beta = \frac{cR}{2}$; $\omega_0^2 = \frac{c}{J}$. $[c] = Nm$; $[R] = \frac{\frac{1}{s}}{Nm} = \frac{1}{Nms}$; $[\beta] = Nm\frac{1}{Nms} = \frac{1}{s}$, $[\omega_0] = \sqrt{\frac{Nm}{kg\,m^2}} = \frac{1}{s}$
e)	$P_C + P_R + P_L = 0$; $\frac{V}{K_V} + R\dot{V} + L_V\ddot{V} = 0$; $\ddot{V} + \frac{R}{L_V}\dot{V} + \frac{1}{L_V K_V}V = 0$; $\beta = \frac{R}{L_V}$; $\omega_0^2 = \frac{1}{L_V K_V}$; $K_V = \frac{\Delta V}{\Delta P}$; $\ddot{V} + 2\beta\dot{V} + \omega_0^2 V = 0$. $[\beta] = \frac{\frac{Pa\,s}{m^3}}{\frac{Pas^2}{m^3}} = \frac{1}{s}$; $[L_V] = \frac{Pa}{\frac{m^3}{s^2}}$, $[K_V] = \frac{m^3}{Pa}$, $[R] = \frac{Pa}{\frac{m^3}{s}}$, $[\omega_0] = \sqrt{\frac{1}{\frac{Pa\,s^2 m^3}{m^3\ Pa}}} = \frac{1}{s}$. Taking into account the ratio n = $\frac{\rho}{M}$V, it is possible to define the law of liquid quantity transformation in the vessels.
f)	$U_C + U_R + U_L = 0$; $\frac{Q}{C} + R\dot{Q} + L_e\ddot{Q} = 0$; $Q + CR\dot{Q} + CL_e\ddot{Q} = 0$; $\ddot{Q} + \frac{R}{L_e}\dot{Q} + \frac{1}{L_e C}Q = 0$; $\beta = \frac{R}{2L_e}$; $\omega_0^2 = \frac{1}{L_e C}$; $\ddot{Q} + 2\beta\dot{Q} + \omega_0^2 Q = 0$.

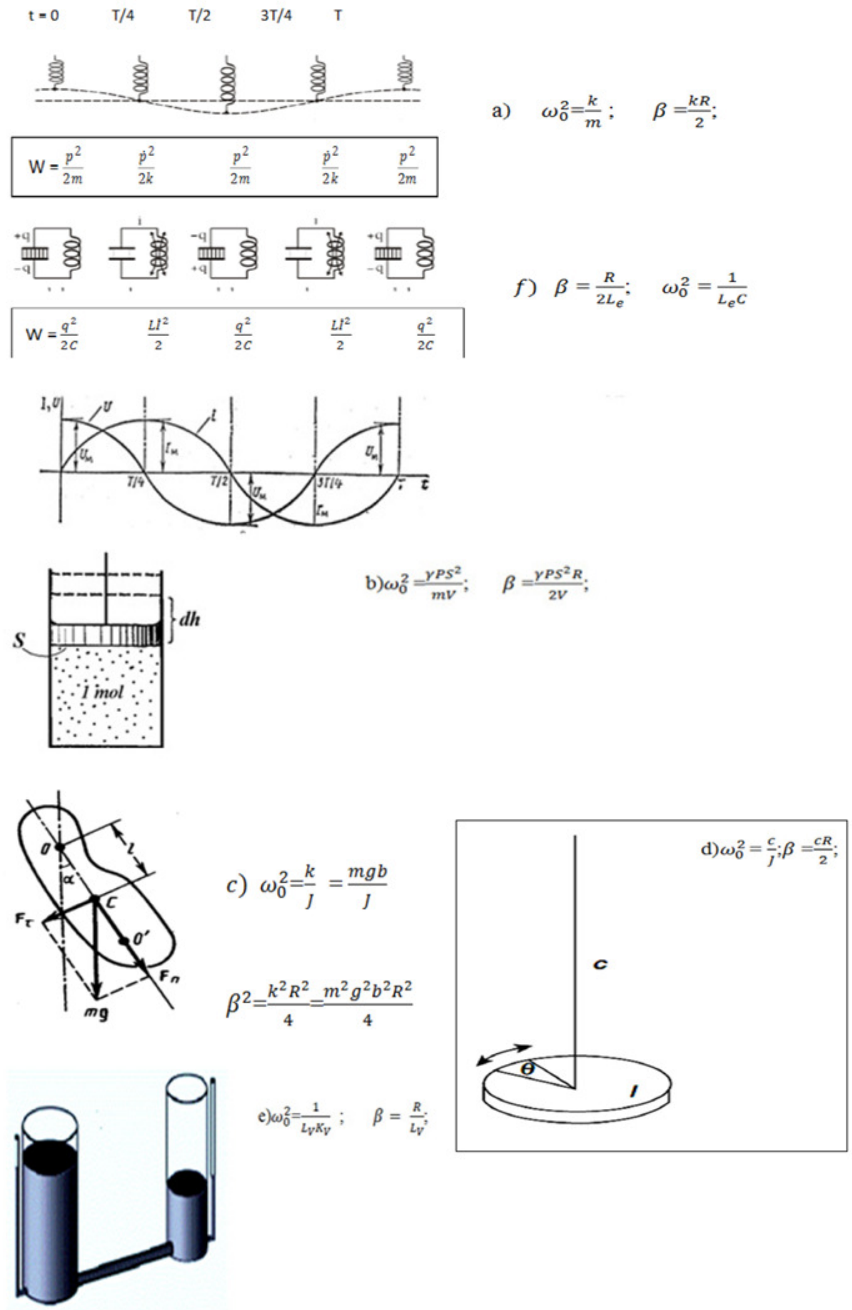

Fig. 2: Oscillatory system diagrams

As described above at the moment time t=0, the capacitive potential difference is completely consumed by the inductive potential difference and the resistive potential difference remains zero. Therefore, the momentum current also becomes zero. At the moment of time when t=T/4, it's vice versa, the capacitive potential differential is completely consumed by resistive potential difference, and the inductive potential difference remains zero. Therefore, at this moment of time, the change in the momentum current becomes zero, and the momentum current becomes maximum.

4. Conclusion

1. In case of linear motion of the body, the extensive property (energy carrier) is the body's momentum. Therefore, the equation of motion, as well as in the electrical form of motion, must be bound by an extensive quantity and its derivatives. In oscillations, where the linear motion is performed (e.g., a spring pendulum), the equation should be written as a function of the momentum and its derivatives, instead of the spatial coordinate *x*. The spatial coordinate *x* cannot be an extensive value in a linear form of the body motion. The spatial coordinate does not include properties inherent to

the body. The division of the body into two parts has no effect on *x*.

2. Taking a spatial coordinate as an extensive quantity (coordinates of state) at the linear shape of body motion may lead to inaccurate results in the analogy to the constructive parameters in oscillatory systems such similarity of mass and inductance. The research performed in the article shows that the mass is an analog of electrical capacity and the elasticity is an analog to inductance. These pairs are characteristics of identical physical properties of various motion forms.

3. The correct definition of natural analogs allows to implement a unified approach; in the analyzed oscillatory systems the laws of conservation and transformation of energy were applied, expressing the transformation of energy through extensive quantities of appropriate forms of motion and their derivatives, thus receiving the same kind of mathematical equations.

4. It has been determined that the oscillatory systems are composed of subsystems with the same properties, and it also allows using a unified approach.

5. A unified approach saves time in the teaching of oscillation processes, allows gaining system knowledge, correctly understanding the processes, generalizing, drawing conclusions and simplifying the process of remembering.

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