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Introducing a new method of calculating mathematical morphology using associative memories

Ali Azimi Kashani [*], Alimohamad Monjezi Nori, Mohsen Ganj, Iman Mosavian

Department of Computer Engineering, Shoushtar Branch, Islamic Azad University, shoushtar, Iran

Abstract: The method presented in this paper involves the application of Auto-Associative lattice memories to defining a type of ordering monitored by LAAM: a special type of ordering h which facilitates effect civil definition of morphological operators that operates on multivariable data. The mathematical morphology aimed at room using multivariable images requires definition of proper orderings which allow us to define elementary morphological operators without erroneous results (such as wrong colors). All the calculations are defined by lattice algebra operators (∨,∧,+), so the LAAM monitored orderings are done much faster than those already presented. They also impose less calculating burden. Furthermore, in the LAAM monitored function h, it is possible to identify the recalling errors based on LAAM for each vexcel.

Key words: *Lattice auto-associative memories; Mathematical and morphology; Multivariable awarding; Vexcel*

1. Introduction

Mathematical morphological was presented by Serra (1984, 1988) and Maragos (1975) as a powerful means of analyzing images. During the last 50 years, mathematical morphology has turned out to be efficient in such fields as processing the form and sight of machines. It has also been utilized in slicing medical images, classifying via remote assessment, identifying millions, analyzing texture, and identifying geometrical shapes. Morphological operators involve scripts between complete lattices. They are denoted by $\mathbb{L}$ or $\mathbb{M}$ and are partially ordered sets in which the infimum and the supremum for each pair are defined. Erosion and dilation are two basic morphological operators. Consider rs-fMRI eight offers the following groups of people: healthy cases (HC), schizophrenic patients with affected hearing, and those without affected hearing (SZAH and SZnAH, respectively). We are going to detect brain lattice differences among these groups. LAAM monitored h ordering is a lattice calculation corresponding with the approaches based on dependent or independent complement analysis. This ordering is used to analyze static state fMRI and searches for low-frequency lactic complement lattices. We use a vexcel BOLP time series to construct a LAAM. This process applies to other vexcels in fMRI four-dimension data. In this state, the function h shows the functional similarities of the brain. After recognizing brain lattices and lattice the map desired from the brain information, the functional relations in the brain are revealed. Using the morphological operators capable of extracting particular features from the vexcels, we can carry out this process.

2. Multivariable ordering

Morphological operators work well with scalar images, but it is not convenient to expend and use them directly to work on multivariable images unless a general order for vector spaces is defined which can also maintain the natural features of morphological operators. For example, an important feature is that the output of erosion and dilation operators should be within the values in the figure, and using them should not lead to the production of any new color. The projects in(Darya Chyzhyk, 2013. J. Serra, 1993) are examples of multivariable ordering that suffer from production of wrong colors. A solution to this problem is to write the multivariable values as scalar through defining reductive ordering (Aptoula and Lefevre, 2007; Angulo 2007). Through an overlapping script of the main data set as a complete lattice, the ordering is defined as $h : X \rightarrow L$ so that the order in the target lattice results from a general order in the source set X. It means $r \leq_h r' \Leftrightarrow h(r) \leq h(r')$

$$x \leq_h y \Leftrightarrow h(x) \leq h(y);\ \forall x, y \in X. \qquad (1)$$

The reduced ordering can be defined based on a monitored classifying system. The values of the discretizer function or deductive probabilities used to assess classes allow for the overlapping h script. A distinction is often considered between foreground and background classes in the size of two standard classes. In the normalization presented in (Angulo, 2007), the monitored ordering h on the non-empty

[*] Corresponding Author.

set $\mathbf{X}$ is an ordering h that realizes $h(b) = \perp, \forall b \in B$ and $h(f) = \top, \forall f \in F$ so that $B, F \subset X$ are subsets of X where $B \cap F = \emptyset$, and $\perp$ and $\top$ stand for the higher and lower elements of the target lattice, respectively. Erosion operators increase parts of the figure near the background, and dilation operators increase parts near the foreground. Since the h functions are not necessarily one-to-one, the resulting h orderings may not offer a general order. While we need to distinguish between the members of the classes equivalent to $l[z] = \{c \in R^n | h(c) = z\}$, to avoid ambiguity, our criteria will be the lexical order.

2.1. Multivariable morphological operators

For a multivariable figure $\{I(p) \in R^n\}_{p \in D_I}$ (where $\mathbf{D_I}$ denotes the spatial area of the figure) and by considering the structural object $\mathbf{S}$, the monitored erosion operator h is defined as $\boldsymbol{\varepsilon}_{\mathbf{h,s}}(\mathbf{I})(\mathbf{p}) = \mathbf{I}(\mathbf{q})\ \mathbf{s.t.}\ \mathbf{I}(\mathbf{q}) = \wedge_{\mathbf{h}}\{\mathbf{I}(\mathbf{s}): \mathbf{s} \in \mathbf{S_p}\}$ where $\wedge_{\mathbf{h}}$ the infimum obtained from the reductive ordering $\leq_{\mathbf{h}}$ of equation 1. $\mathbf{S_p}$ is this structural elements interpreted at pixel $\mathbf{p}$ location. The monitored dilation operator h is defined as $\boldsymbol{\delta}_{\mathbf{h,s}}(\mathbf{I})(\mathbf{p}) = \mathbf{I}(\mathbf{q})\ \mathbf{s.t.}\ \mathbf{I}(\mathbf{q}) = \vee_{\mathbf{h}}\{\mathbf{I}(\mathbf{s}): \mathbf{s} \in \mathbf{S_p}\}$ in which $\vee_{\mathbf{h}}$ is the supremum according to equation 1. The morphological gradient for images with scalar values is defined as the difference between erosions and dilations of image $\mathbf{I}$ with the structural element $\mathbf{S}$ So that $\mathbf{g_S}(\mathbf{I}) = \boldsymbol{\delta}_{\mathbf{S}}(\mathbf{I}) - \boldsymbol{\varepsilon}_{\mathbf{S}}(\mathbf{I})$. In multivariable images, the monitored morphological gradient of h is defined as the monitored erosion of h, i.e. $\boldsymbol{\varepsilon}_{\mathbf{h,S}}(\mathbf{I})$, and the dilation $\boldsymbol{\delta}_{\mathbf{h,S}}(\mathbf{I})$ is as follows:

$$g_{h,S}(I) = h\left(\delta_{h,S}(I)\right) - h(\varepsilon_{h,S}(I)) \qquad (2)$$

The LAAM h script is defined as Chebyshev distance between the main pattern vector and the recalling resulting from LAAM. In (Ann K. Shinn, Justin T. Baker, Bruce M. Cohen, and Dost Ongur, 2013), this distance has been used as the classification factor of the nearest neighbor based lattice**.** In the proposed method, this distance serves as the monitored classification factor. If $x \in \mathbb{R}^n$ is a sample data vector and $X = \{x_i\}_{i=1}^{K}$, $x_i \in R^n$ is a non-empty training set, for all $i = 1, \ldots, K$, the LAAM h script is calculated through:

$$h_X(c) = d_c(x^{\#}, x) \qquad (3)$$

where $\mathbf{x}^{\#} \in \mathbb{R}^{\mathbf{n}}$ is the answer of recalling of the LAAM dilation of $\mathbf{M_{XX}}$ to the input of vector $\mathbf{x}$ as $\mathbf{x}_{\mathbf{M}}^{\#} = \mathbf{M_{XX}} \barwedge \mathbf{x}$. Instead of it, we may use the erosion recalling of memory $\mathbf{W_{XX}}$ as $\mathbf{x}_{\mathbf{W}}^{\#} = \mathbf{W_{XX}} \bar{\vee} \mathbf{x}$. The function $\mathbf{d_c(a, b)}$ denotes the Chebyshev distance between two vectors, which is calculated by using the largest absolute difference between the vector components as $h_X(c) = d_c(x^{\#}, x)$

2.2. And LAAMs h script

Consider a training set $\mathbf{X}$: A monitored foreground LAAM h ordering denoted by $\leq_{\mathbf{X}}$ is defined based on the LAAM h script of equation 3 as:

$$\forall x, y \in R^n, x \leq_X y \Leftrightarrow h_X(X) \leq h_X(y) \qquad (4)$$

The monitored foreground LAAM h ordering produces a complete lattice $\mathbb{L}_{\mathbf{X}}$ in which the lower element $\perp_X = 0$corresponds to the set of fixed points M_{XX} and W_{XX} , i.e. $h(x) = \perp_X$ for u $x \in \notin(X)$. On the other hand, the upper element is $\top_X = +\infty$.

2.3. Background/Foreground LAAM h script

To construct a background/foreground LAAM monitored h ordering, we need a training set B for the background and a training set F for the foreground separately. The foreground LAAM h the script defined in equation 4 is applied separately to the date by using the training sets B and F , and h_B and h_F scripts are obtained, respectively. Here, we build a background/foreground (B/F) h script by combining h_B and $h_F$with a single h script and name it $h_r(x)$ as follows:

$$h_r(x) = h_F(x) - h_B(x) \qquad (5)$$

The value is positive for $x \in \notin(B)$ and negative for $x \in \notin(F)$. Therefore, this function is employed as a separating function so that $h_r(x) > 0$ $\mathbf{h_r(x) > 0}$ denotes the pixels in background class and $h_r(x) < 0$refers to foreground class pixels. The police in which $h_r(x) = 0$ occurs are considered as decision criteria. The corresponding monitored h ordering denoted by $\leq_r$ is defined as:

$$\forall x, y \in R^n, x \leq_r y \Leftrightarrow h_r(x) \leq h_r(y)) \qquad (6)$$

The form of background/foreground LAAM h script is a complete $\mathbb{L}_r$ lattice whose lower and upper elements are $\perp = -\infty$ and $\top = +\infty$, respectively. This h script does not precisely comply with the standards given in because we have $h_r(b) \neq \perp$for $b \in B$ and also $h_r(f) \neq \top$ for $f \in F$. However, the obtained orderings and the resulting morphological operators submit appropriate results.

3. The proposed method and the patterns under study

This section presents two experiments conducted on rs-fMRI data. In the first experiments, the patterns consistent with each group of people are collectively analyzed. In this analysis, we calculate the average of each group's spatial rs-fMRI data. Then, we calculate the Tanimoto coefficient for the identified lattices to obtain the appropriate threshold levels to find the brain's functional lattices. In the first experiment, we use reduced orderings functional scripts to find the brain lattices. We particularly studied brain lattices supplied from specific brain sites to detect the differences among certain people. The results revealed that this method can find and show the cluster differences which are functionally related through identifying patients suffering from hearing disorder. We also performed our experiment with LAAM h function in both foreground and background states. In the second experiment, the grouping results for each group of the population are presented. To perform the grouping, we obtain date features as follows: first, we build the LAAM monitored h function, which provides a reduced ordering script related to the left Heschl's gyrus. Pearson coeff of correlation between the values of function h and the variable related to each grouping in each vexcel location allow us to locate the vexcels of higher information value. Using the values of function h, we construct features values in these locations. The results obtained from k-NN separators show that this method can create an accurate distinction among different populations.

3.1. Results of the experiments on rs-fMI

What is presented in this section includes the results of the experiments on static-state fMRI data in which 28 healthy cases (HC) and two groups of get out from schizophrenic patients (26 with affected hearing (SZAH) and 14 with non-affected hearing (SZnAH)) were selected. Tanimoto coefficient: Tanimoto coefficients are used to find the similarity of the lattices found in order to use the appropriate threshold level to introduce the lattices found. In two sets A and B, Tanimoto coefficient states the similarity level of these two sets through the cardinal proportion of their intersection and union

$$\mathrm{T(A,B)} = \frac{|\mathrm{A}\cap\mathrm{B}|}{|\mathrm{A}\cup\mathrm{B}|}, \qquad (7)$$

This equation measures the overlap amount of the two sets. Tanimoto coefficient is normalized within $\mathrm{T} \in [0,1]$ interval. If there is no similarity, then $\mathrm{T} = 0$, and if there is complete conformity, then $\mathrm{T} = 1$. When comparing the results of slicing the images, the sets will be those very areas of the figure. Pearson correlation: Pearson correlation coeff is calculated as follows:

$$\mathrm{r} = \frac{\mathrm{n}(\sum \mathrm{xy})-(\sum \mathrm{x})(\sum \mathrm{y})}{\sqrt{[\mathrm{n}\sum \mathrm{x}^2-(\sum \mathrm{x})^2][\mathrm{n}\sum \mathrm{y}^2-(\sum \mathrm{y})^2]}}. \qquad (8)$$

Where $\mathrm{r} \in [-1,1]$, and $\mathrm{r} = 1$ means perfect positive correlation between the two variables, and $\mathrm{r} = -1$ denotes absolutely negative correlation. In our field of study, Pearson correlation assesses the labels of a pre-calculated class, and fMRI neural links using function h.

3.2. First experiment

The purpose of conducting this experiment is to discover brain lattices capable of distinguishing between healthy people, schizophrenic patients with affected hearing, and those with non-affected hearing. The results show that LAAM monitored h ordering recognizes completely different brain lattices depending on the person under study by using one single h function built from a variety of selected vexcels. The lattice locations correlated with a particular vexcel, especially from a part of the cortex related to hearing, show the effects that pertain to hearing disorder. vexcel time series used to construct LAAM have been extracted from a healthy person data. We created the map corresponding to the application of LAAM monitored h function ones for the healthy case and once for the schizophrenic patient. We illustrated the lattice by using the vexcels with the highest similarity according to h function. Finally, we depicted the findings obtained from the maximum points of top-hat conversion and presented the results by means of the brain scripts created by one-way and background/foreground orderings.

3.2.1. One-way monitored h ordering

In this section, we investigated the results of two samples, one in the front part of the brain and the other in part associated with hearing. Figure 1a shows the selected vexcel's location near the front part. We arbitrarily chose it from the gray matter. Figure 1b depicts the network for his vexcel in a healthy person data with no further processing. Figure 1c shows the lattice for patients with affected hearing. A number of pertinent vexcels have occurred more in them than in the healthy person and in the patient with non-affected hearing. Figure 1d illustrates these findings for the patient with non-affected hearing. It can be inferred that in this state, the lattice is much smaller than the patients with affected hearing. Figure 2a shows the vexcel location in the hearing area of the brain. Therefore, what this method offers is creation of brain lattices dependent upon the selected vexcels. The main points in the results is that the apart from the vexcel used to construct $\mathbf{h_x}$, there is great lack of correspondence between the lattice found for schizophrenic patient with affected hearing and that for the one with non-affected hearing. The result of applying top-hat filter that results in some localization and is shown in figures 4a and 4b confirms this conclusion. These calculations have been done based on h script age created by the points in figure 1a and 2a.

3.2.2. Background/foreground monitored ordering

We used the vexcels in the brain's white matter (WM) in lattice area and the lattice liquid in the brain lattice as the background points. The vexcels in the gray matter (GM) area also served as the foreground. The vexcels were used to calculate $\mathbf{h_r}$ script. Figures 3a-3d shows the selected points. The meaning of the other figures is also the same as the previously presented figures. The result of applying top-hat filter shown in figures 5a-5d and obtained from h script of figure 3a-3d confirm this conclusion.

3.3. Second experiment

First, we perform pre-processing operations on rs-fMRI data to ensure that all the fMRI values are balanced and homogeneous with spatial normal structural T1waiting data. Then, we calculate the background/foreground LAAM h script based on the normalized data in which the background data are related to lattice liquid in the vexcels of the brain lattice, and the foreground data, based on(S. Velasco-Forero and J. Angulo,2011), include a selective collection of vexcels of the left Heschl's gyrus. We utilize h scripts and a vexcel-oriented method all over the brain to calculate Pearson coefficient with a deductive variable that groups the subjects.

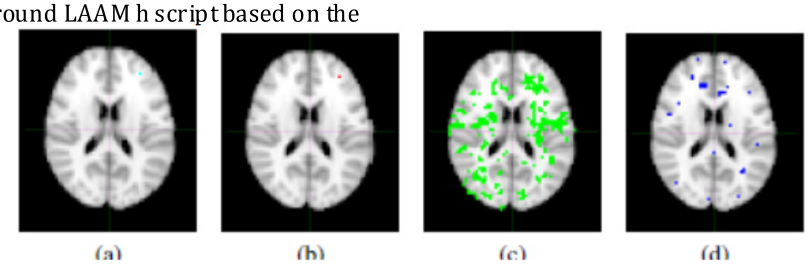


Fig. 1: points of brain's front part. a. vexcel locations in a healthy case, b. corresponding vexcels lattice in the healthy case, c) schizophrenic patient with affected hearing, d. schizophrenic patient with non-affected hearing

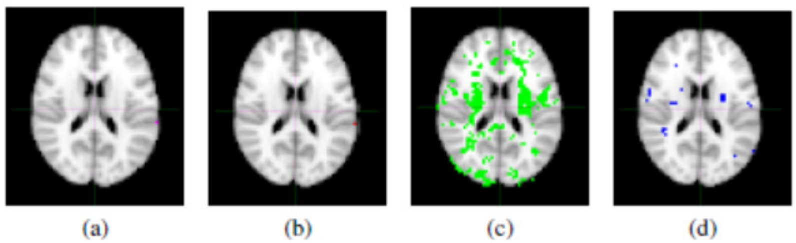


Fig. 2: points of hearing cortex in the brain. a. vexcel locations in a healthy case, b. corresponding vexcels lattice in the healthy case, c) schizophrenic patient with affected hearing, d. schizophrenic patient with non-affected hearing

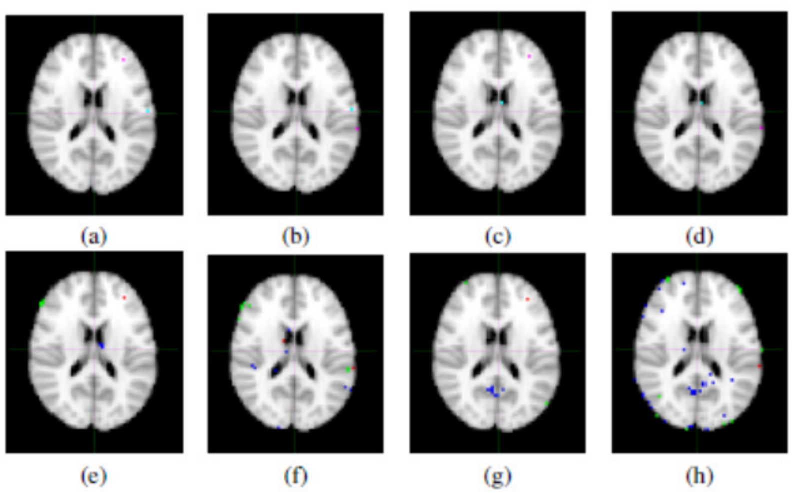


Figure 3: points related to the vexcels type. a. background from the white matter and foreground from the great master of the front area, b. background from the white matter and foreground from the great master of the hearing area, c. background from the lattice liquid of the brain lattice and foreground from the gray matter of the front part of the brain, d. background from the lattice liquid of the brain lattice and foreground from the gray matter of the hearing area, which is used for $\mathbf{h_r}$ ordering. Blue and pink colors depict background and foreground vexcels, respectively, e. vexcel locations in a healthy case, f. corresponding vexcels lattice in a healthy case, g. schizophrenic patient with affected hearing, h. schizophrenic patient with non-affected hearing

Then, we obtain a correlated script. The locating of vexcels with the highest correlation coefficient absolute values defines masks that can extract features. The masks are used to create features vectors of people's h scripts. The masks can locate the indexes of the fingers with possible medical value. The features vectors are employed to conduct grouping experiments by applying a 10-layer validation method. In order to obtain basic results, we use grouping factors k-NN. We take it for granted that the results are accurate in order to verify the index values obtained by means of the feature masks.

The results of grouping in figure 6 indicate the results of grouping experiments based on the distinctions among possible class pairs: healthy cases vs. schizophrenic patients (HC vs. Schiz), vs. patience with non-affected hearing (HC vs. nAH), vs. patience with affected hearing (HC vs. AH), and between two groups of patients with affected and on non-affected hearing (nAH vs. AH). The color bars show the magnitude of the feature vector created from the location of vexcels having the highest Pearson correlation coefficient absolute value.

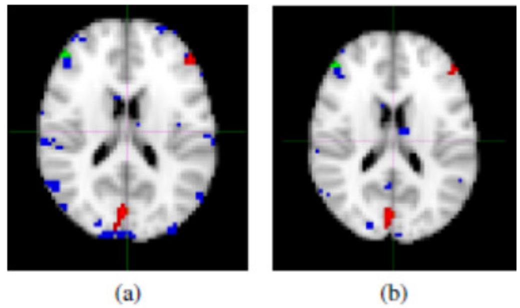


Fig. 4: locating through top-hat filter calculated according to $\mathbf{h_x}$ ordering derived from the points in figures 1a and 1b for figures a and b. Their red, green, and drew vexcels are devoted to healthy cases, schizophrenic patients with affected hearing, and those with non-affected hearing, respectively

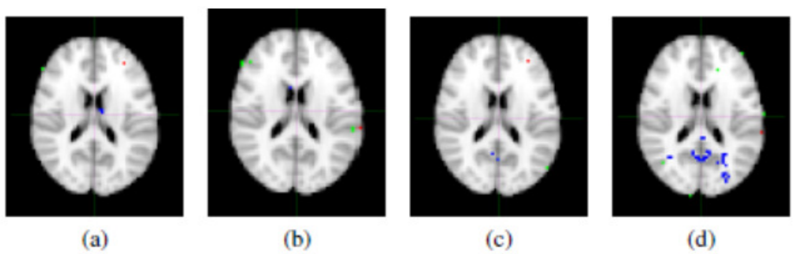
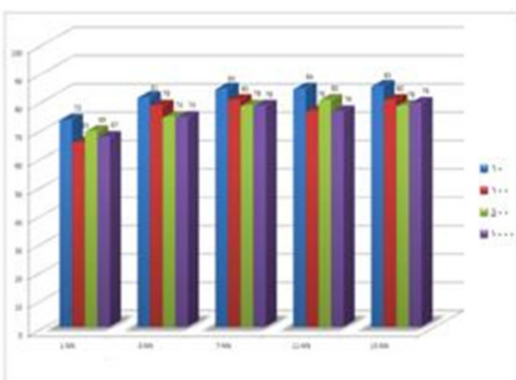
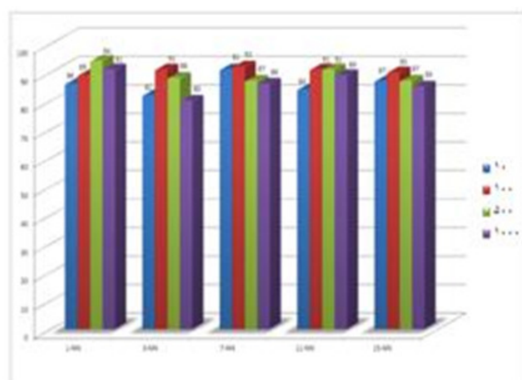


Fig. 5: locating to top-hat filter: $\mathbf{h_r}$ ordering is done based on point pairs of background/foreground in figures 3a-3d



Healthy case vs. schizophrenic patient with non-affected hearing



schizophrenic patient with non-affected hearing vs. schizophrenic patient with affected hearing

Fig. 6: the highest grouping accuracy was obtained by 10 times of iterations of identity assessment algorithm for k-NN classifier and $k = 1, 3, 7, 11, 15$. The bars' colors indicate different numbers of the obtained features

4. Conclusion

In this paper, a multivariable mathematical morphology using lattice calculation techniques was introduced. By using the LAAM model error, measured by Chebyshev distance as a reduced ordering h script, we defined first different monitored h orderings and then the operators and filters of mathematical morphology. To be more accurate, we constructed a foreground ordering and also a LAAM monitored foreground/background ordering. The main merit of these definitions is the stability and adaptability of morphological filters and operators. The proposed method requires definition of the vexcels set to analyze fMRI data. Of course, hypotheses and common statistical techniques are of no use in this method, i.e. the method is not generally model-dependent. Moreover, the proposed method relies only on the lattice calculational operators.

Therefore, the operators needed to create further intelligence include only min (finding minimum value), Max (finding maximum value), and summation. This makes the problem produce fewer errors than other mathematical methods. In the first experiment, the initial method showed that it could submit some lattice locations with their strong differences and consistent with the groups of people under study. This type of distinction can be used in machine learning methods. In the second experiment, LAAM monitored background/foreground h script on rs-fMRI submits the features necessary to identify the potential indexes of schizophrenic patients. It also distinguishes between patients with and without hearing disorder via calculating Pearson correlation coefficient with a deductive variable. These indexes are assessed in terms of the accuracy of the grouping factors obtained from feature vectors derived from selected vexcels. The results of this grouping is satisfactory, particularly in distinguishing between healthy cases and non-affected-hearing patients. It is possible to obtain further results by applying more intricate groupings to the data. It is based on this multivariable mathematical morphology that you can employ morphological filters to select the features.

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