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The mathematical model of portfolio optimal size (Tehran exchange market)

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Abstract: Following Markowitz's model in early 1950 ,containing risk concept and presenting the theory of "capital asset pricing model" and analyzing the risk and risky asset into systematic and unsystematic by Sharpe ,some researchers in different financial markets by using the concept "well diversification "tried to do same researches to drive the optimal size of a portfolio. In this research as well as mentioned researchers, we tried to derive the optimal size of a portfolio through "well diversification "and with the help of Tehran exchange market's data (during 1991 -2011). The concept of optimal portfolio is a kind of portfolio which the amount of risk is nearly in systematic level and it is impossible to be eliminating through well diversification. The result of the research by the researchers show that the optimal size of a well-diversified portfolio in Tehran exchange market includes a portfolio with at least 14 common stocks in which the amount of systematic risk is about 0.1.

Key words: *Capital asset pricing model; Systematic risk; Unsystematic risk; The optimal size of a portfolio; Well diversification*

1. Introduction

Harry Markowitz was the first researcher who stated the concept of risk in a quantitative form in early 1950 and with the help of the mentioned concept; He succeeded in deriving nonlinear mean-variance efficient frontier.

Later on, William Sharpe ,by introducing some assumptions such as Market Portfolio and the existence of a free risk asset with R_f pure interest rate managed to change the nonlinear efficient frontier into a linear form known as Capital Market Line (CML) in the financial literature .(Sharpe, 1963 -1964; Markowitz, 1952-1959)

The result of Sharpe and Lintner's efforts not only led to deriving liner efficient frontier but also resulted in the separation of the risk of risky assets into systematic and unsystematic risk. (Lintner, 1965)

Following the mentioned researches, some researchers in financial markets such as whitmore, Wagner, Evans and Archer, Brennan and Epps tried to derive the optimal size of a Portfolio .i.e. a Portfolio which the amount of risk is in systematic level.

The above mentioned studies show that in the U.S exchange market by regarding the concept of "well diversification "nearly all Portfolios with more than 10 stocks include only systematic risk.

Since the size of an optimal Portfolio is very important for investors, in this article we aimed to derive the optimal size of a Portfolio in Tehran exchange market during the period of 1991-2011.

The importance of the issue refers to the fact that if potential investors reach this assurance that the risk of their chosen Portfolio is nearly in systematic level, then the probability of gaining their expected Portfolio rate of return will be high.

2. Literature review

We have learnt that CAPM model is the development of Markowitz's model from a specific view and is the first theory in capital asset pricing models (Fama, 1996-2004) which was presented by Sharpe and Lintner. In fact, the study of portfolio theory by Harry Markowitz which demonstrates the relationship between risk and return was first published in 1952 and updated in 1959, provided the grounds for emergence of the first theory in capital asset pricing (CAPM) by William Sharpe. Sharpe by choosing specific assumptions succeeded in developing Markowitz's nonlinear efficient frontier to linear efficient frontier, which is known as Capital Market Line (CML) in the financial literature. An important and remarkable starting point in the CAPM model is the assumption of the existence of market portfolio (M).

According to this assumption, market portfolio not only exists but also is computable and lies on Markowitz's efficient frontier. In fact, the existence of such assumption enables us to derive the maximum amount for Sharpe's ratio.

We also learnt that Sharpe and Lintner succeeded in deriving linear efficient frontier and separating the risk of a risky asset into systematic and unsystematic risk.

In addition, some other researchers in financial market world such as Whitmore, Wagner, Evans and

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Archer, Brennan and Epps made an attempt to derive the optimal size of a Portfolio model presentation.

In order to achieve the mentioned aim i.e. the way of determining the optimal size of a well-diversified portfolio, the historical data related to stocks which are annually in access, will be used.

Let's imaging R^k represents the calculated return related to stock **K** .according to the definition we have:

$$R_i^K = \frac{P_{i+1}^k + d_i^k}{P_i^k} \qquad (1)$$

So that:

P: represents the price of a stock.

d: represents the amount of dividend paid

i: represents time.

Now by using the concept of geometrical mean , the average total return of stock **k** in the studied period will be as follows:

$$\bar{R}^k = exp\left(\frac{1}{n}\sum_i^n \log_e \frac{P_{i+1}^K + d_i^k}{P_i^k}\right) \qquad (2)$$

Also, with the help of relation (3) the risk of stock **k** will be calculated as follows:

$$SD^k = \left[\frac{1}{n-1}\sum_{i=1}^n (\log_e \overline{R_k} - \log_e R_i^k)^2\right]^{\frac{1}{2}} \qquad (3)$$

Now, according to the obtained relationships (1 to 3)16 Portfolios are selected as follows.

First, one common stock of all available stocks is randomly chosen in Tehran Stocks Market and with the help of relation (1 to 3) the related amounts of risk and return are calculated. The name of this Portfolio is called a Portfolio with the size of 1. next , two common stocks of all available stocks which exclude the first stock are randomly chosen in Tehran's stocks markets and again with the help of relation (1 to 3)the related amounts of risk and return are calculated. The name of this Portfolio is called a Portfolio with the size of 2.the above mentioned process is repeated for Portfolios with the size of 3 to 16.

In order to calculate the average return of a Portfolio with the size of m, relation (4) is used in every period as follows:

$$\bar{R}_\iota = \frac{1}{m}\sum_{k=1}^{m} R_i^k \quad : \quad m = 1,2,3,\dots,16 \qquad (4)$$

Later, in order to calculate the total return of a Portfolio in the studied period, relation (5) is used as follows:

$$\overline{R_p} = exp\left(\frac{1}{n}\sum_{i=1}^{n} \log_e \bar{R}_\iota\right) \qquad (5)$$

Also, in order to calculate the risk of a Portfolio in the studied period, relation (6) is used as follows:

$$SD_p = \left[\frac{1}{n-1}\sum_{i=1}^{n} (\log_e \overline{R_p} - \log_e \bar{R}_\iota)^2\right]^{\frac{1}{2}} \qquad (6)$$

In addition, in order to increase the efficiency of the presented model, we tried to repeat the Process for five times for each desired size of a Portfolio. So 80 Portfolios were selected (i.e. 5×16).

At the end, after deriving the average amounts of the risk of a Portfolio with the size of 1 to 16 we tried to estimate the regression model between the size of a Portfolio and the average risk.

3. Model selection

In order to select the best regression model, we compared 8 regression models which are presented in Tables No 1 and 2. According to the Tables 1 and 2, it can easily be seen that inverse and power regression models can be regarded as the best regression models.

Since the two above mentioned models have rather the same significance level, we will compare their F amounts. Under such conditions, power regression model will be a better model.

The selected regression model will be as follows:

$$Y_i = AX_i^B + \varepsilon_i \qquad (7)$$

So that:

Y_i , X_i and ε_i represent the size of Portfolio, the average risk of Portfolio size and error term respectively.

Table 1: Inverse regression model

Size of portfolio	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Observes	Standard deviation															
1	0.240	0.200	0.180	0.180	0.110	0.111	0.100	0.115	0.125	0.116	0.131	0.115	0.110	0.115	0.071	0.110
2	0.160	0.220	0.160	0.150	0.130	0.113	0.130	0.122	0.145	0.114	0.108	0.135	0.114	0.100	0.129	0.093
3	0.200	0.180	0.140	0.140	0.160	0.137	0.127	0.114	0.110	0.121	0.122	0.112	0.112	0.108	0.100	0.097
4	0.260	0.170	0.100	0.120	0.120	0.169	0.123	0.125	0.101	0.102	0.116	0.100	0.111	0.110	0.118	0.110
5	0.290	0.180	0.170	0.170	0.180	0.130	0.120	0.124	0.114	0.137	0.113	0.112	0.115	0.102	0.112	0.120
Average of Standard deviation	0.23	0.19	0.15	0.14	0.123	0.122	0.12	0.118	0.119	0.118	0.118	0.114	0.112	0.107	0.106	0.106

Source: research findings

Table 2: Power regression model

Equation	Model Summary					Parameter Estimates			
	R Square	F	df1	df2	Sig.	Constant	b1	b2	b3
Linear	0.675	29.132	1	14	0.000	0.182	-0.006		
Logarithmic	0.925	172.785	1	14	0.000	0.211	-0.041		
Inverse	0.960	331.739	1	14	0.000	0.104	0.137		
Quadratic	0.878	46.681	2	13	0.000	0.222	-0.019		
Cubic	0.963	104.197	3	12	0.000	0.258	-0.041	0.001	
Compound	0.769	46.671	1	14	0.000	0.182	0.961	0.004	0.000
Power	0.966	393.891	1	14	0.000	0.217	-0.269		
S	0.913	146.281	1	14	0.000	-2.224	0.856		
Growth	0.769	46.671	1	14	0.000	-1.706	-0.040		
Exponential	0.769	46.671	1	14	0.000	0.182	-0.040		
Logistic	0.769	46.671	1	14	0.000	5.504	1.041		

Source: research findings

3.1. Proposed model estimation

In Order to estimate the parameters of regression proposed model, proposed model will be estimated by using ordinary least squares (OLS) and with the help of the Table 1 data. The results of the estimation are easily shown in Fig. 1 and Tables 3 and 4.

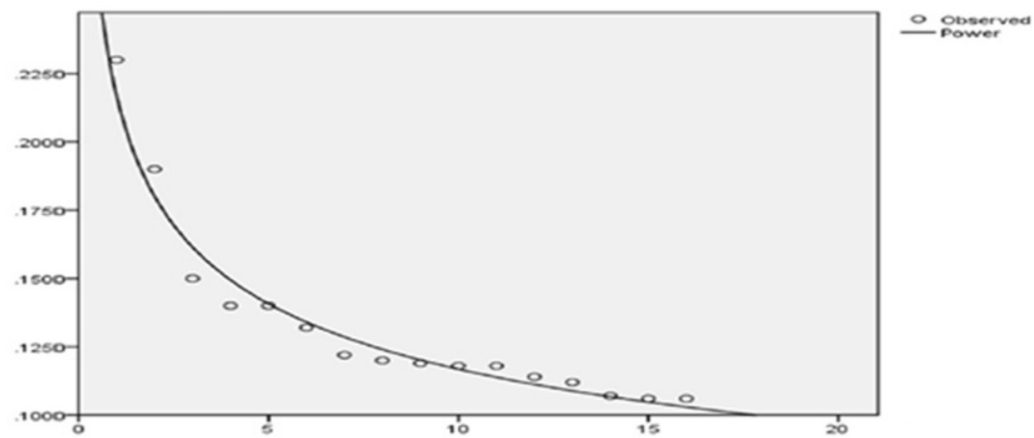

Fig. 1: Standard deviation

Table 3: ANOVA

	Sum of Squares	df	Mean Square	F	Sig.
Regression	0.675	1	0.675	393.891	0.000
Residual	0.024	14	0.002		
Total	0.699	15			

Source: research findings

Later on, according to table (3) the significance hypothesis test of regression model will be as follows:

H_o: Power relation exists between two variables

H_1: Power relation doesn't exist between two variables

According to the last column of table (3) the exact amount of significance, equals,

$1.1935430204057872 \times 10^{-11}$

And it means that with the probability of 99.9% null Hypothesis will be verified and in contrast, alternative Hypothesis will be rejected.

Also, according to table (4), the significance Hypothesis test of A and B parameters will be as follows:

H_0: A= 0 H_1: A≠ 0

H_0: B= 0 H_1: B≠ 0

So that the exact amounts of Significance are:

$3.643294889856826\times10^{-15}$

$1.1935430204057872\times10^{-11}$

Respectively which means that alternative Hypothesis are verified with the probability of 99.9%.

According to the available data in Table 4, the proposed model Regression will be as follows:

$$Y_i = 0.217X_i^{-0.269} \quad (8)$$

Table 4

	Unstandardized Coefficients		Standardized Coefficients		
	B	Std. Error	Beta	t	Sig.
In(portfoilio size)	-0.269	0.014	-0.983	-19.847	0.000
(Constant)	0.217	0.006		35.783	0.000

Source: research findings

Also, According to relation (8) and figure .1 it can be easily seen that if investors of Tehran's stock exchange selected a Portfolio containing more than 14 common stocks with the use of well diversification principle, then the risk of selected Portfolio would be in systematic level which the amount would be equal to 0.1 approximately.

4. Conclusion

In this research, by using historical data related to price and dividend, we tried to derive the mathematical model of the optimal size of a well-diversified Portfolio in Tehran exchange market during the period of 1991-2011.

In this research, it was shown that well diversification resulted in decreasing the whole risk of a portfolio to the systematic level and also, it was shown that the optimal size of a well-diversified portfolio in Tehran exchange Market included a portfolio with at least 14 common stocks.

The importance of creating a well-diversified portfolio refers to the fact that the probability of gaining the expected return by an investor can be increased.

Finally, by choosing a random sample including 50 investors, it was shown that 62% of the mentioned investors didn't Pay attention to "well diversification principle" also 89% of them didn't regard the optimal size of a portfolio and 51% didn't paid attention to neither of them.

In other words these two groups of investors made the two mentioned mistakes which can impose irreparable loss to them.

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