WALIA journal 31(S2): 170-175, 2015
Available online at www.Waliaj.com
ISSN 1026-3861

Modeling and solving the location-inventory problem in supply chain with NSGA-II algorithm

Mohammad Ekramifard, Marjan Mohammadjafari *

Department of industrial Engineering, Kerman Branch, Islamic Azad University, Kerman, Iran

Abstract: Simultaneous consideration of location-inventory and supply costs results in better answers in comparison with considering them in separate times as Shen et al. (2003) have proven this based on calculations. In this article, a new mathematical model has been proposed to the multi-objective problem of location-inventory in supply chain. The first objective function seeks to minimize these costs simultaneously while the second objective function seeks to maximize the coverage extent to the customers' demand by distribution centers. Due to the complexity of this problem, an evolutionary multi-objective non-dominated sorting genetic algorithm (NSGA-II) has been presented to solve it at an appropriate time. After having applied this algorithm, its output results has been expressed in large sizes and then the results have been compared with the output results of the exact method in small size, and the data have been analyzed. Performing these steps of analyzing, it is observed that application of the proposed algorithm is acceptable. Finally, the conclusion and suggestions for further research have been offered.

Key words: *Supply chain; Location; Inventory control; Meta-heuristic algorithm; Covering*

1. Introduction

Nowadays severe competition in global markets forced researchers to design and manage the supply chain more efficiently. Supply chain management is generally classified in three main groups: strategic, tactical and operational. In making these decisions it is important to locate a facility system (Lu and Bostel, 2006).

Being usually as part of planning for hierarchical logistic system, the problem of locating, in fact, is a strategic subject because in many logistics, managers have to make decisions on the locating depots to include warehouses, factories and distribution centers, and also on the allocation of customers to each service center (Berman and Wang, 2006).

The NSGA-II algorithm has been applied in this paper in order to achieve the optimal combination of problem's parameters and optimization objectives of meta-heuristic algorithm. One of the practical objectives of the problem is to present and solve a practical model for supply chain management model. By applying a few changes, the presented model has the possibility of using in industrial environments such as cellphone antenna's, tower cranes and spoiled able food distribution centers location.

This article's combinational problem of location and inventory is a multi-objective, non-linear, integer and therefore a formidable issue. Generally, heuristic and meta-heuristic methods have been used for solving and optimizing this kind of problems. Using meta-heuristic methods for solving such problems, have improved the acceptable solution practically by increasing the size of the problem, while traditional methods lose their efficiency. For example Liao et al. (2011) used NSGA-II algorithm for optimizing an integrated model of location and inventory.

This paper is organized as follows: the literature review of the problem has been presented in Section 2. The model has been introduced completely in Section 3 while the NSGA-II algorithm is proposed in Section 4 in order to solve the presented problem. The computational results are in Section 5, and the conclusions and the future researches' scope have been included in Section 6.

1.1. Literature review

Facility location problem is one of the most important fields of operation research problems in which many books and papers have been published (Melo et al, 2009). Supply chain management is one of the logistics areas that have attracted so much attention. In fact, supply chain management independently developed from operation research and gradually entered in supply chain management (Chopra and Meindl, 2007). Using mechanical facility models suggested in supply chain management and thus was created a rich field of application.

Facility location problems are used for designing distribution networks that consists of setting location for facility resources and assigning them to potential customers. One example for facility location problem is to set location of production sites, assign potential customers to these sites and set retailers to warehouses.

* Corresponding Author.

Facility and assignment problem can be defined as a set of potential dots for creating service centers and estimating customers' demands while each group of customers should be assigned to one of the service centers, which has a fix cost, in order to supply their demands. Facility location consists of setting service centers and assigning it to costumers in order to reduce costs or reach specific level of coverage for all customers (Tuzun and Burke, 1999).

Locating policies, including Long-term or medium-term policies and inventory policies including Long-term or medium-term policies in distribution networks are considered but the effect of facility location policies on inventories' cost are obvious, because by changing the assigned demands to providers, inventory costs also change. In addition suggested models minimize both inventory and location costs.

It has been shown computationally by Shen et al (2003) that using both inventory and location costs is better than separating them.

Location-inventory models can be classified according to the following:

- models with limited storage capacity
- Multi-commodity models with a limited number of storage
- Location-inventory models with constrained service providers capacities.

One of the most prominent problems in location problems is to maximum coverage location problems (MCLP). This problem is trying to maximize coverage demand dots by constant number of facilities through considering a distance or critical specified time for coverage. In coverage location problem, there is no obligation to cover all dots. Church and Revvelle (1974) proposed MCLP. Megiddo et al. (1983) defined MCLP problem for tree network and suggested an algorithm for solving it.

Renewable planning approach, by Zeleny (1982) under the title De Novo was introduced to solve resource allocation problems at first. Despite the usual mathematical models in which the resource capacity is fixed and known in advance, in De Novo approach capacity levels have been considered as a decision variable that affect objective function.

2. Problem formulation

2.1. Problem description and assumptions

In this paper, hybrid location-inventory problem in supply chain has been purposed to define the location of distribution centers, assign customers to the centers and determine the optimal values of the inventory control system. In this research a three-stage (one supplier, many distribution centers, and many customers) supply-chain network is considered.

The main assumptions used in the problem are:

1. Each customer is assigned to each product up to a distribution center.
2. Each customer must be assigned to a distribution center.
3. Each customer demand is possible, independent and conformed normal distribution.
4. Inventory system of DCs, is (r,Q).
5. Lead time is possible and conformed normal distribution.
6. The capacity of supplier is unlimited.
7. There is a direct transfer of product from supplier to customer.

2.2. Notations

The following notation is used here:

Parameters

G_k: The minimum order quantity for direct application of the main supplier for product (k).

h_j^k: The annual maintenance cost per unit of product (k) in the center (j).

f_j : Fixed cost of creating a distribution center (j).

O_j^k: Fixed cost of ordering product (k) in the center (j).

w_j: Distance to the center of supplier (j).

c_j^k: Transportation cost per unit of product (k) from the supplier to the center (j), per unit distance.

L_j^k: Average lead time product (k) to the center (j), per day.

D_i^k: Average daily demand of customer (i), of product (k).

$Z_{1-\alpha}$: Level of service

d_{ij}: Distance customer (i) to center (j).

v_{ij}^k: Transportation cost per unit of product (k) for customer (i) from the center of (j), per unit distance.

t: Number of days per year.

E_j: The cost per unit of capacity creating in the center (j).

S_j^k: The maximum length for maximal coverage of products (k) in the center (j).

T_j^k: The maximum length for Partial covering of products (k) in the center (j).

b_{ij}^k: Covering center (j) to customer (i) of product (k).

σ_i^k: Standard deviation of daily demand of customer (i) of product (k).

$\sigma_{L_j}^k$: Standard deviation of lead time product (k) to the center (j), per day.

Covering Function:

$$b_{ij}^k = \begin{cases} 1 & d_{ij} \le S_j^k \\ f^k(d_{ij}) & S_j^k < d_{ij} \le T_j^k \quad ; \quad (0 < f^k(d_{ij}) < 1) \qquad f^k(d_{ij}) = \left(\frac{(T_j^k - d_{ij})}{(T_j^k - S_j^k)}\right) \\ 0 & d_{ij} \ge T_j^k \end{cases} \tag{1}$$

Indices:

I: Customers J: Distribution center (j=1: main supplier) K: Products

Variables:

$$x_{ij}^{k} = \begin{cases} 1 & customer\ i\ for\ product\ k\ to\ distribution\ center\ j \\ 0 & otherwise \end{cases}$$

$$y_j = \begin{cases} 1 & distribution\ center\ j\ is\ active \\ 0 & otherwise \end{cases}$$

P_j: Number of capacity creating in the center (j) Q_j^k: The order of product (k) in the distribution center (j)

SS_j^k: Safety stock of product (k) in the distribution center (j)

$$SS_j^k = Z_{1-\alpha}\sqrt{L_j^k.\left(\sum_{i\in I}\left((\sigma_i^k)^2(x_{ij}^k)^2(b_{ij}^k)^2\right)\right) + \sigma_{l_j^k}^2.\left(\sum_{i\in I}(x_{ij}^k b_{ij}^k D_i^k)\right)^2} \quad (2)$$

2.3. Mathematical formulation

Two-objective mathematical model presented:

$$Min\ Z1 = \left[\sum_{j\in J}(f_j Y_j)\right] + \left[\sum_{j\in J}(E_j P_j Y_j)\right] + \left[\sum_{k\in K}\sum_{j\in J}\sqrt{2\left(\left(\sum_{i\in I}(D_i^k x_{ij}^k t b_{ij}^k)\right).(h_j^k o_j^k)\right)}\right] \quad (3)$$

$$+\left[\sum_{k\in K}\sum_{j\in J}\left(h_j^k.Z_{1-\alpha}\sqrt{\left(L_j^k.\left(\sum_{i\in I}\left((\sigma_i^k)^2(x_{ij}^k)^2(b_{ij}^k)^2\right)\right) + \sigma_{l_j^k}^2.\left(\sum_{i\in I}(x_{ij}^k b_{ij}^k D_i^k)\right)^2\right)}\right)\right]$$

$$+\left[\sum_{k\in K}\sum_{j\in J}\sum_{i\in I}\left((w_j c_j^k).(D_i^k.x_{ij}^k.t.b_{ij}^k)\right)\right] + \left[\sum_{k\in K}\sum_{j\in J}\sum_{i\in I}\left((d_{ij} v_{ij}^k).(D_i^k.x_{ij}^k.t.b_{ij}^k)\right)\right]$$

$$Max\ Z2 = \left[\sum_{k\in K}\sum_{j\in J}\sum_{i\in I}(D_i^k.x_{ij}^k.t.b_{ij}^k)\right] \quad (4)$$

Subject to:

$$\sum_{j\in R_i^k} x_{ij}^k = 1; \qquad i\in I, k\in K \quad (5)$$

$$x_{ij}^k \le Y_j; \qquad i\in I, k\in K, j\in R_i^k \quad (6)$$

$$P_j \le (Y_j.M); \qquad j\in J \quad (7)$$

$$\left[\left(\sum_{k\in K}\sum_{i\in I}(D_i^k.x_{ij}^k.t.b_{ij}^k)\right) + \left(\sum_{k\in K}\left(Z_{1-\alpha}\sqrt{\left(L_j^k.\left(\sum_{i\in I}\left((\sigma_i^k)^2(x_{ij}^k)^2(b_{ij}^k)^2\right)\right) + \sigma_{l_j^k}^2.\left(\sum_{i\in I}(x_{ij}^k b_{ij}^k D_i^k)\right)^2\right)}\right)\right)\right] \le (Y_j P_j); \quad j\in J \quad (8)$$

$$D_i^k \,.\, x_{i1}^k \geq G_k . x_{i1}^k \qquad (9)$$

$$Y_j \in \{0,1\}; \qquad j \in J, n \in N \qquad (10)$$

$$x_{ij}^k \in \{0,1\}; \qquad i \in I, k \in K, j \in R_i \qquad (11)$$

$$P_j \geq 0 \; and \; INT \; ; \; j \in J \qquad (12)$$

The objective of function (3) is to minimize the supply chain that consists of: location costs, inventory costs and transportation costs. The objective of function (4) is to maximize the covering of DCs. Constraints (5) ensure that each customer in relation to any product can be allocated directly to one of the DCs. Constraints (6) ensure that when one of the DCs are activated, the customer's products can be allocated. Constraints (7) ensure that the capacity of inactivated DCs is zero. Constraints (8) are related to the maximum capacity of each of the activated distribution centers. Constraints (9) ensure that if $D_i^k \geq G_k$, the customer is allowed for ordering to the main supplier. Constraints (10) and (11) provide the binary condition. Constraints (12) ensure that Capacity DCs are integers.

3. Designing NSGA-II algorithm for presented model

3.1. Solution procedure

One of the important components of the algorithm is the results of solution procedure. Shown in Fig. 1 are solution procedures.

	K=1					K=2					…
j \ i	1	2	3	…	C	1	2	3	…	C	…
1	0	0	1		0	0	0	1		0	
2	1	1	0		1	0	1	0		0	
3	0	0	0		0	0	0	0		1	
⋮	⋮										⋮
Dc	0	0	0		0	1	0	0		0	

Fig. 1: Solution procedures

3.2. Generating initial solutions

For generating initial solutions a three-dimensional matrix has been used. First of all a random integer number (r) generated between [1, dc] and if it is possible the element A (i, r, k) is equaled 1. It means that the assignment of product (k) of customer (i) to distribution center (r) is possible if the distribution center is the main supplier. The conditions are $D_i^k \geq G_k$ و $b(i,r,k) > 0$, else $b(i,r,k) > 0$.

3.3. Crossover operator

In this paper the uniform crossover has been used. For setting each column of children, one coin thrown out. If lion appears, the column related to the child one from P_1 and the column related to the child two from P_2 are selected, otherwise vice versa. (P_1 And P_2 are two selected parents for crossover operator).

3.4. Mutation operator

The flip mutation has been used in this paper. The length of each chromosome is $k \times m$ and each gene is selected by the probability of $\frac{1}{k \times m}$. A random integer number generated between [1, dc] and selected one (j) and if it is possible the number is equaled 1 and other distribution centers are equaled 0.

4. Calculation results

In this section, first results of 10 small sizes samples have been compared with the results of the algorithm, and input parameters have been adjusted via Taghuci Method, and the proposed algorithm for 15 larger size samples has been solved.

4.1. Designing sample of problem

25 sample of problem have been solved in small and large sizes. Each problem depends on 3 factors: number of products, customers and DCs.

4.2. Eternal parameters

Input dates are shown in the table 1(You and Grossmann, 2008).

Table 1: Input parameter

parameter	measure	parameter	measure
v_{ij}^k	Random number between [0.2,0.3]	f_j	Random number between [75,125]
t	250 days	h_j^k	Random number between [5,15]
E_j	Random number between [5,15]	G_k	Random number between [150,200]
S_j^k	Random number between [60,120]	O_j^k	Random number between [7,12]
T_j^k	Random number between [90,180]	c_j^k	Random number between [1,10]
σ_i^k	Random number between [25,80]	L_j^k	Random number between [5,10]
$\sigma_{L_j}^k$	A half lead time	D_i^k	Random number between [50,200]

Customer zones, DCs and supplier were generated from a uniform distribution over a square with side 100(Amiri, 2006).

4.3. Adjusting Taguchi parameter

NSGA-II Algorithm has 4 input parameters. In order to adjust these parameters, the DOE of the Taguchi method in 9 levels has been used. For each factor 3 levels have been considered. Rang of these parameters have been shown in Table 2.

Table 2: Rang of parameters

Multi-objective Algorithms	Parameters Algorithm	Parameter range	Small size			Large size		
			small	Medium	High	Small	Medium	High
NSGA-II	nPop	(25-200)	25	60	95	130	165	200
	nIter	(50-150)	50	70	90	110	130	150
	Pcrossover	(0.6-0.99)	0.6	0.67	0.75	0.83	0.91	0.99
	Pmutation	(0.01-0.4)	0.01	0.08	0.16	0.24	0.32	0.4

According to Fig 2 and 3 adjusted parameters have been highlighted in Table 2.

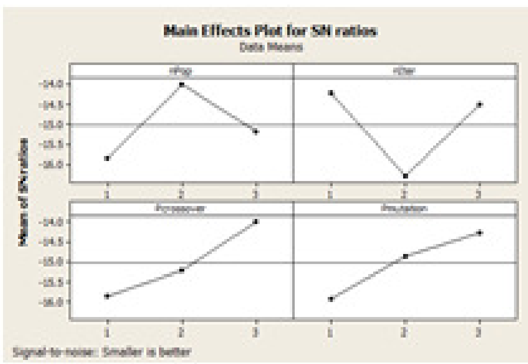

Fig. 2: small size

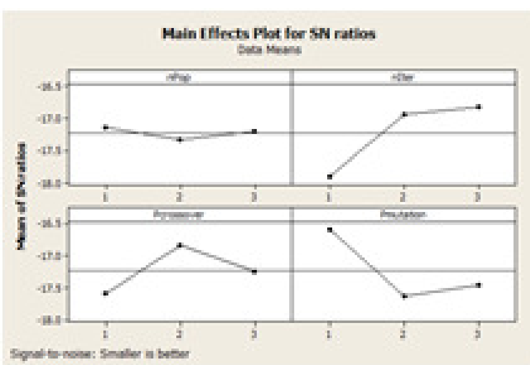

Fig.3: Large size

4.4. Choose the best indicator of the proposed algorithm

Taguchi testing algorithm is based on Pareto points. There are two main purposes: divergence in the first iterations of the solution and the convergence of iterations to Pareto answer. Among the criteria for assessing the performance of the multi-objective algorithm, The MID and time as response have been used (sarrafha et al, 2014).

At the calculation of MID, one of the objectives is to minimizie and the other one is to maximize, so the objectives don't have the same units. Therefore the objectives should be normalized. The second objective function changed to $\frac{1}{covering}$ and for normalization has been used $\frac{f_i(x)}{f_i(x^*)}$ (Marler and Arora, 2005). So the objectives converted as below: Then MID has calculated and here the ideal point is (0, 0).

$$\min \; cost\ i \;,\; \max \; covering\ i \quad \forall i; Pareto\ frontier$$

The results of comparison between the exact and the meta-heuristic approaches are shown in Table 3.

Table 3: Comparison of exact and meta-heuristic approach

Size	Run problem	I	J	K	GAMS Time (sec)	NSGA-II Time (sec)
Small size problem	1	3	2	2	30.18	4.1311
	2	4	2	2	50.68	4.0087
	3	4	2	3	100.23	4.0957
	4	5	2	3	292.49	4.1467
	5	6	2	3	- - ->900	4.2897
	6	7	4	4	--------	4.0235
	7	7	6	4	--------	4.0643
	8	7	8	5	--------	4.2489
	9	8	8	5	--------	4.1161
	10	9	8	5	--------	4.1234

Large size problems have been solved with NSGA-II algorithm and the results have been shown in Table 4.

Table 4: Results of large size problem

Size	Run problem	I	J	K	NSGA-II	
					Time	MID
Large size problem	1	20	15	10	55.4004	4.6595
	2	25	17	10	59.2471	4.3172
	3	30	18	10	61.9200	5.5715
	4	35	19	11	64.0620	5.2733
	5	40	20	11	68.9633	4.2785
	6	45	21	11	70.4659	3.9869
	7	45	22	12	77.5174	3.1397
	8	50	23	12	79.7244	4.0096
	9	50	24	12	81.7383	3.3346
	10	60	26	13	90.5070	3.4812
	11	65	26	13	94.5958	2.8463
	12	70	28	13	99.0117	3.4545
	13	75	28	14	105.3212	4.5364
	14	80	30	14	113.1742	3.9126
	15	100	30	15	127.8644	2.7397

5. Conclusions and Future trends

In this paper a new hybrid mathematical location-inventory problem has been presented. The objectives of this problem are to minimize the supply chain costs and to maximize the covering of DCs. This model is complicated so it's kind of NP hard problems. Therefore meta-heuristic NSGA-II algorithm has been proposed. Designing several small and large problems then solved and compared these problems with MID and time index. The results show that the proposed algorithm had an acceptable performance. At the end for the future research, it can be implemented in competitive environments. For future studies also, discussing model with fuzzy data, would be highly appropriate. In this paper, considering a period of time change, the consumer's location and demand for them in various periods could be other topics for future researches.

References

Amiri A., 2006, Design a distribution network in a supply chain system: Formulation and efficient solution procedure, European Journal of Operational Research, vol. 171, pp. 567-576.

Berman, O., Wang, Q., 2006, Inbound Logistics Planning: Minimizing Transportation and Inventory Cost, Transportation Science, vol. 40, pp. 287-299.

Chopra, S., and Meindl, P., 2007, Supply chain management. Strategy, planning and operation. Das Summa Summarum des Management, 265-275.

Church, R, L,. Revelle, C., 1974, The maximal covering location problem, Regional Science Association, vol. 32, pp. 101-118.

Fengqi You and Ignacio E. Grossmann, 2008, mixed-integer Nonlinear Programming Model and Algorithms for Large-Scale Supply Chain Design with Stochastic inventory Management, Ind. Eng. Chem. Res., vol. 47, No. 20, pp. 7802-7817.

Liao S. H., Hsieh C. L., Lai P. J., 2011, An evolutionary approach for multi-objective optimization of the integrated location- inventory distribution network problem in vendor-managed inventory, Expert System with Applications, vol. 38, pp. 6768-6776.

Lu, Z., Bostel, N., 2006, a facility location model for logistics systems including reverse flows: The case of remanufacturing activities, Computers and Operations Research, vol. 34, pp. 299-323.

Megiddo, N., Zemel, E., Hakimi, S. L., 1983, The maximum coverage location problem, SIAM Journal on Algebraic Discrete M ethods, vol. 4, pp. 253-261.

Melo, M. T., Nickel, S., and Saldanha-Da-Gama, F., 2009, Facility Location and Supply Chain management-A review. European Journal of Operational Research, 196(2), 401-412.

Sarrafha K., Kazemi A., Alinezhad A., 2014, A Multi-objective Evolutionary Approach for Integrated Production-Distribution Planning Problem in Supply Chain Network, Journal of Optimization in Industrial Engineering, vol. 14, pp. 89-102.

Shen, Z. J. M., Coullard, C., Daskin, M. S., 2003, A Joint Location-Inventory Model, Transportation Science, vol. 37, pp. 40-55.

Timothy, R., Marler, J. S. Arora, 2005, Function-transformation methods for multiobjective optimization. *Engineering Optimization*, vol. 37(6), pp.551–570.

Tuzun, D., Burke, L. I., 1999, A two-phase tabu search approach to the location routing problem, European Journal of Operational Research, vol. 116, pp. 87-99.

Zeleny, M., 1982, Multiple Criteria Decision Making, McGraw Hill Book Company, New York.