WALIA journal 31(S6): 12-16, 2015
Available online at www.Waliaj.com
ISSN 1026-3861

Drafting exponential equations and their solutions

Akan Alpyssov [1,*], Zhazira Mukanova [1], Gulzhan Jarassova [2], Rashid Mussaibekov [3], Lazzat Aldibayeva [4]

[1]*Pavlodar State Pedagogical Institute, 140002, Pavlodar, Mira Street, 60, Kazakhstan*
[2]*Pavlodar State University named after S. Toraigyrov, 140000, Pavlodar, Lomov Street, 64, Kazakhstan*
[3]*Kokshetau State University named after Sh. Ualikhanov, 030000, Kazakhstan*
[4]*Kazakh National Agrarian University, 020000, Almaty, Kazakhstan*

Abstract: The article describes the process of compiling exponential equations and their solutions. This method contributes to the formation of mathematical thinking and the development of cognitive abilities of students. In solving problems used: logical laws revealed new connections between mathematical facts. It promotes creativity in the study of mathematics. In the process of exponential equations to develop creative activities and formed the logic of thinking.

Key words: *Solution of exponential equations; The method of inverse action; Knowledge generation; Conversion; Logic thinking*

1. Introduction

Analysis of the works of Armin Mahmoudi [1], Karl Wesley Kosko and Anderson Norton [2], Pulfer, J. D and Whitehead, M. A. [3], Clute P. [4], Sakenov, D. Zh. [5], George L. Trigg [6], Daane C.J., Judy G. and Tina S. [7], David K. Pugalee [8], Page Starr and Vladimir Rokhlin [9], Hembree R. [10], Alpysov A., Mukanova Z. [11], Cheryl A. Lubinski and Albert D. Otto [12], Kai Velten [13], Peter D. Miller, Nicholas M. Ercolani, Igor M. Krichever and C. David Levermore [14], Kellah Edens and Ellen Potter [15] showed that the process of drafting exponential equations promotes formation of mathematical thinking and the development of cognitive abilities of students.

The equation of the form

$$a^{f(x)} = c.$$

Call the standard exponential equation. Giving parameters different numerical values, we obtain the specific equation. For example, assuming

$$a = 3,\ c = 2,\ f(x) = \sqrt{x},$$

we obtain the equation $3^{\sqrt{x}} = 2$. This equation is solved by the method of inverse action:

$$\log_3 3^{\sqrt{x}} = \log_3 2,\ \sqrt{x} = \log_3 2,$$

$$\left(\sqrt{x}\right)^2 = (\log_3 2)^2,\ x = \log_3^2 2.$$

Formulation of the problem for the first case can be written as:

Example 1. $a^{f(x)} = c \ \rightarrow\ f(x) = \log_a c$

The above example is a solution to this problem. Formulation of the problem for the first case can be written as:

Example 2. $a^{x^2} = c \ \rightarrow\ x^2 = \log_a c$

The requirements of the tasks, the information needed for the structure a and c parameters as well $f(x)$. First of all, the number should be expressed as a power of the base a with an odd index. So specification has led to an equation of the form $2^{x^2} = 32$.

Example 2. 1 $a^{f(x)} = c \ \rightarrow\ x = k$.

Decision: The task set so that solution must be represented as a number, and then a common fraction. The information in the request is given in the form of a literal expression, the value of which is determined by them. Aliasing requirements problem stems from the fact that the numbers on the right side, is relevant both to the ground level a, and to f. It is easy to show, for example, the number represented in the form of a degree from the base equal a, i.e. $c = a^k$. The equation becomes:

$$a^{f(x)} = a^k \ \rightarrow\ f(x) = k.$$

Parameter does not depend on x and f. Therefore, as it can take a number of values. We hold that for value can be a prime number:

$$a = 5,$$

And for the exponent take an arbitrary number, say k = 3, then $c = 5^3 = 125$. Because, in effect dividing the requirement specified, for f take effect multiplying.

For Example,

$$f(x) = 6x.$$

---
* Corresponding Author.

Thus, the action in the index and in demand linked mutually inverse relationship, i.e., the action taken in the demand for basic, but for the action off to take back to him. So has specific equation:

$$5^{6x} = 125.$$

$$a^{2f(x)} + pa^{f(x)} + q = 0$$

$$a^{f(x)} = y$$

$$y^2 + py + q = 0.$$

$$a^{f(x)} = c.$$

Thus, the structure of the standard exponential equation is related, on the one hand, a three-term exponential equation, the other - a square. The exponential function can be contained in an open or closed mold. In the latter case it is necessary to look for the structure of the exponential function for replacement. Furthermore, a quadratic equation may have a solution, and the exponential equation has.

$$c = a^{3k+2}$$

a=2, κ=1.

$$2^{x^2} = 32,$$

$$x = \pm\sqrt{5}.$$

The requirement to compile a specific exponential equation can be expressed as follows: exponential equation must be an integer or fractional numerical solution of a solution or a periodic solution. To meet these requirements is selected corresponding structure function f (x). In other words, in the process of drawing up the equation a person, in particular compares the structure of the numeric data with the structure and function in the exponent doing other mental operations. This is one of the important factors. It promotes the development of thinking. We now consider the three-term equation in which exponential function is contained in the structure of the quadratic equation. Exponential function may hit in the open or in a closed form. The combination of these forms may be affected by the existence of a decision as a square, and exponential equations. First of all, we express the quadratic equation through its roots.

$$y^2 - (y_1 + y_2)y - y_1 \cdot y_2 = 0$$

$$a^{2f(x)} - (y_1 + y_2)a^{f(x)} - y_1 \cdot y_2 = 0$$

It is an open form of exponential equation. Dividing both sides of this equation to $a^{f(x)}$, obtain an equation

$$a^{f(x)} - (y_1 + y_2) - y_1 \cdot y_2 \cdot a^{-f(x)} = 0$$

Here, the structure of the quadratic equation given in a latent form.

2. Methods

The logic of research is caused by use of system of the methods complementing each other: the theoretical analysis philosophical, psychology and pedagogical, and natural-science literature on a problem; modeling; studying of pedagogical experience; diagnostics, questioning, purposeful pedagogical supervision, interviewing; conversations, oral and written polls, testing; pedagogical experiment; introspection and self-assessment students of the activity; statistical processing of materials of research; analysis of results of skilled and experimental work.

3. Main part

It is necessary to create an exponential equation which has no solution:

Example 1.

$$a^{f(x)} - (y_1 + y_2) - y_1 \cdot y_2 \cdot a^{-f(x)} = 0 \quad \rightarrow \quad x = \{0\}$$

Decision: The structure contains a trinomial equation solving a quadratic equation. If the quadratic equation will only have a negative decision, the singling out of here exponential equations will have solutions.

$$y_1 = -1,\ y_2 = -3.$$

$$a^{f(x)} + 4 + 3 \cdot a^{-f(x)} = 0$$

$$y^2 - (y_1 + y_2)y - y_1 \cdot y_2 = 0$$

$$y = a^{f(x)},$$

$$a^{2f(x)} - (y_1 + y_2)a^{f(x)} - y_1 \cdot y_2 = 0$$

$$a^{f(x)} - (y_1 + y_2) - y_1 \cdot y_2 \cdot a^{-f(x)} = 0$$

Now draw up threefold exponential equation, from then you can extract only one standard exponential equation. This problem in mathematical language can be written as:

Example 2.

$$a^{f(x)} - (y_1 + y_2) - y_1 \cdot y_2 \cdot a^{-f(x)} = 0 \quad \rightarrow \quad a^{f(x)} = c \ \ (c > 0) \text{-?}$$

Decision: The request to the problem is said to make only one standard exponential equation that meets this requirement

$$a^{f(x)} - 3 - 4 \cdot a^{-f(x)} = 0$$

$$a^{f(x)} = y,$$

$$y - 3 - 4 \cdot y^{-1} = 0$$

$$y^2 - 3y - 4 = 0,$$

$$y_1 = -1,\ y_2 = 4,$$

$$a^{f(x)} = 4.$$

The problem is solved, but the specification of the structure of the standard exponential equation is completed.

We formulate as a separate task.

Example 2. 1.

$$a^{f(x)} = 4 \rightarrow x = 0, \text{-?}$$

Decision:

$$a = 4.$$

$4^{f(x)} = 4^1 \rightarrow f(x) = 1$.

The request states that the decision should be a periodic number - 0, (c) = V / 9. Action to reverse the division is multiplication. So the question arises: at what number should be multiplied x. In the top ten, only two prime numbers 3 and 7, for which the ordinary fraction 1/3 and 1/7 are converted to decimal periodic fraction. For example, the fraction 1/7 is equal to the period of six figures: 1/7 = 0 (142857). Hence, x is multiplied by 3. The whole of the batch number will be zero if when = 1. The structure of the index was defined: f (x) = 3x. The standard exponential equation and the three-term exponential equation are of the form:

$4^{3x} = 4$, $4^{3x} - 3 - 4^{-3x} = 0$.

Example 2. 2.

$a^{f(x)} - (y_1 + y_2) - y_1 \cdot y_2 \cdot a^{-f(x)} = 0 \rightarrow$

$a^{f(x)} = y_1$, $a^{f(x)} = y_2$

Decision: The standard exponential equation exists when both roots of the quadratic equation are positive.

$y_1 = 3$, $y_2 = 8/3$.

$3a^{f(x)} - 17 + 8a^{-f(x)} \cdot 3 = 0$,

$a^{f(x)} = 3$, $a^{f(x)} = 5$.

To specify the structure of the standard exponential equations formulate problems in mathematical language.

Example 2. 2. 1. $a^{f(x)} = 3 \rightarrow$ *a, f* $\rightarrow$ $x = \frac{\sqrt{5}}{2}$ -?

Example 2. 2. 2. $a^{\varphi(x)} = \frac{8}{3} \rightarrow$ *a,* $\varphi \rightarrow$ $x = -0{,}25$ -?

Decision: The standard exponential equation is solvable if the base of the power of both sides of the same, so this equation can be written in the form $3^{f(x)} = 3$.

f(x)=1.

*f(x)=f*x.*

$f \cdot x = 1$

$f \cdot \sqrt{5}/2 = 1$.

$f = 2/\sqrt{5}$.

$3^{\frac{2}{\sqrt{5}}x} = 3$; $3^{\frac{2}{\sqrt{5}}x+1} - 17 + 8 \cdot 3^{1-\frac{2}{\sqrt{5}}x} = 0$

$a^{2\varphi(x)} - (y_1 + y_2)a^{\varphi(x)} - y_1 \cdot y_2 = 0$

Base degrees are an integer. Did the foundation of a fractional number, *a = p / b.*

$$\left(\frac{p}{b}\right)^{2\varphi(x)} - (y_1 + y_2)\left(\frac{p}{b}\right)^{\varphi(x)} - y_1 \cdot y_2 = 0$$

$b^{2\varphi(x)}$

$$p^{2\varphi(x)} - (y_1 + y_2)b^{\varphi(x)}p^{\varphi(x)} - y_1 \cdot y_2 \cdot b^{2\varphi(x)} = 0$$

$$p^{2\varphi(x)} - (y_1 + y_2)(bp)^{\varphi(x)} - y_1 \cdot y_2 \cdot b^{2\varphi(x)} = 0$$

$y_1 = -2$, $y_2 = -5$,

$p^{2\varphi(x)} + 7(bp)^{\varphi(x)} + 10 \cdot 2^{2\varphi(x)} = 0$,

$y_1 = -3$, $y_2 = 7/2$,

$p^{2\varphi(x)} + 0{,}5(bp)^{\varphi(x)} + 10{,}5 \cdot 2^{2\varphi(x)} = 0$

$\left(\frac{p}{b}\right)^{\varphi(x)} = \frac{7}{2}$

If both roots of the quadratic equation are positive numbers, we get the generalized three-term equation. From it there are two standard exponential equations. By entering into the structure of the quadratic equation exponential function with a parameter we create a block of three-term exponential equations. Now select from this block specific equation. To determine the function must be set in the index structure of square solutions, as the number on the right of the standard equation due to its decision.

Example 3.

$$p^{2\varphi(x)} + 0{,}5(bp)^{\varphi(x)} + 10{,}5 \cdot 2^{2\varphi(x)} = 0 \qquad (7)$$

$\rightarrow \left(\frac{p}{b}\right)^{\varphi(x)} = \frac{7}{2}$, when *x = 8.*

Decision: Those foundation degrees in the presence of structural similarity must be identical, it remains in force. The number 7/2 is the root of the quadratic equation. Consequently, the structure of the standard part of the equation is specified. Solving method of inverse action.

$$\left(\frac{7}{2}\right)^{\varphi(x)} = \frac{7}{2} \rightarrow \log_{7/2}\left(\frac{7}{2}\right)^{\varphi(x)} = \log_{7/2}\left(\frac{7}{2}\right) \rightarrow \varphi(x) = 1$$

$\varphi(x) = 1 \rightarrow$ *x=8,*

x=8 $\rightarrow \varphi(x)$ *-?*

Decision: $x = 8 \rightarrow x = 2^3 \rightarrow \sqrt[3]{x} = \sqrt[3]{8} \rightarrow$

$\sqrt[3]{x} = \sqrt[3]{2^3} \rightarrow \sqrt[3]{x} = 2 \rightarrow \frac{\sqrt[3]{x}}{2} = 1$

$$\left.\begin{array}{l}\varphi(x) = 1 \\ \frac{\sqrt[3]{x}}{2} = 1\end{array}\right\} \rightarrow \varphi(x) = \frac{\sqrt[3]{x}}{2}$$

$$7^{\sqrt[3]{x}} - 0{,}5 \cdot (14)^{\frac{\sqrt[3]{x}}{2}} - 10{,}5 \cdot 2^{\frac{\sqrt[3]{x}}{2}} = 0$$

$$p^{2\varphi(x)} - (y_1 + y_2)(bp)^{\varphi(x)} - y_1 \cdot y_2 \cdot b^{2\varphi(x)} = 0$$

$y_1 = 5$, $y_2 = 3/2$,

$$p^{2\varphi(x)} - 6,5(bp)^{\varphi(x)} + 7,5 \cdot b^{2\varphi(x)} = 0$$

From it there are two standard equations. We formulate them.

Example 3.1. $p^{2\varphi(x)} - 6,5(bp)^{\varphi(x)} + 7,5 \cdot b^{2\varphi(x)} = 0 \rightarrow$

$\left(\frac{p}{b}\right)^{\varphi(x)} = \frac{5}{2}$ when *x=11.*

Example 3.2. $p^{2\varphi(x)} - 6,5(bp)^{\varphi(x)} + 7,5 \cdot b^{2\varphi(x)} = 0 \rightarrow$

$\left(\frac{p}{b}\right)^{\varphi(x)} = 5$ when $x = 1 - \log_5 2$.

Decision. We solve the logarithm, because our standard of equation 5 and P / B - are relatively prime, and the decision of the standard equation is expressed by the logarithm.

In a similar line of reasoning was partly concretized standard equation of the form: $\left(\frac{5}{2}\right)^{\varphi(x)} = \frac{5}{2}$, the logarithm of both sides of the bottom 5/2 obtain $\varphi(x) = 1$.

x=11 $\rightarrow \varphi(x)$ -?

$$x + 1 = 12 \rightarrow \frac{x+1}{12} = 1$$

$$\left.\begin{matrix} \varphi(x) = 1 \\ \frac{x+1}{12} = 1 \end{matrix}\right\} \rightarrow \varphi(x) = \frac{x+1}{12}$$

$$5^{\frac{x+1}{6}} - 6,5(10)^{\frac{x+1}{12}} + 7,5 \cdot 2^{\frac{x+1}{12}} = 0$$

4. Conclusion

Thus, mathematics is an abstract science, so students without learning to think abstractly, cannot form their mathematical abilities. Among mathematicians (Armin Mahmoudi [1], Karl Wesley Kosko and Anderson Norton [2], Pulfer, J. D and Whitehead, M. A. [3], Clute P. [4], Sakenov, D. Zh. [5], George L. Trigg [6], Daane C.J., Judy G. and Tina S. [7], David K. Pugalee [8], Page Starr and Vladimir Rokhlin [9], Hembree R. [10], Alpysov A., Mukanova Z. [11], Cheryl A. Lubinski and Albert D. Otto [12], Kai Velten [13], Peter D. Miller, Nicholas M. Ercolani, Igor M. Krichever and C. David Levermore [14], Kellah Edens and Ellen Potter [15]) formed the view that the more math problems will be solved, thus the abstract ideas by themselves will develop. To date, observed the presence of a negative impression of this opinion. Of course, without solving the problem cannot form abstract thinking. It is a necessary condition, but it is not sufficient. Therefore, instead of three or four to solve various problems, it is useful to solve a problem in several ways.

References

Alpysov A., Mukanova Z., 2014. Making up of the logarithmic equations by complication method. Life Sci J, 11(5s):140-144.

Armin Mahmoudi, 2011. Mathematics Performance and Academic Hardiness, Mathematics Anxiety in Adolescence. World Rural Observations, 3(3):122-125.

Cheryl A. Lubinski and Albert D. Otto, 2010. Preparing K-8 Preservice Teachers in a Content Course for Standards-Based Mathematics Pedagogy. School Science and Mathematics, 104(7): 336–350. DOI: 10.1111/j.1949-8594.2004.tb18252.x

Clute P., 1984. Mathematics Anxiety, Instructional Method and Achievement in a Survey Course in College Mathematics. J Research in Mathematics Education, 5: 50-58.

Daane C.J., Judy G. and Tina S., 1986. “Mathematics Anxiety And Learning Styles: What Is The Relationship In The Elementary Pre Service Teachers?”. J School Science and Mathematics: 84-88.

David K. Pugalee , 2010. Writing, Mathematics, and Metacognition: Looking for Connections Through Students’ Work in Mathematical Problem Solving. School Science and Mathematics, 101(5): 236–245. DOI: 10.1111/j.1949- 8594.2001.tb18026.x.

George L. Trigg, 2006. Analytic Methods. Mathematical Tools for Physicists, 1: 33–81.

Hembree R 1990. The Nature, Effects, and Relief of Mathematics Anxiety. J Research in Mathematics Education, 21(1):33–46.

Kai Velten, 2009. Principles of Mathematical Modeling. Mathematical Modeling and Simulation: Introduction for Scientists and Engineers, 1:1–46.

Karl Wesley Kosko and Anderson Norton, 2012. Relationships Between the Process Standards: Process Elicited Through Letter Writing Between Preservice Teachers and High School Mathematics Students. School Science and Mathematics, 112 (6): 340–348.

Kellah Edens and Ellen Potter, 2008. How Students “Unpack” the Structure of a Word Problem: Graphic Representations and Problem Solving. School Science and Mathematics, 108(5): 184–196.

Page Starr and Vladimir Rokhlin, 2006. On the numerical solution of two-point boundary value problems II. Communications on Pure and Applied Mathematics, 47(8): 1117–1159. DOI: 10.1002/cpa.3160470806.

Peter D. Miller, Nicholas M. Ercolani, Igor M. Krichever and C. David Levermore, 2010. Finite genus solutions to the Ablowitz-Ladik equations. Communications on Pure and Applied

Mathematics, 48(12): 1369–1440. DOI: 10.1002/cpa.3160481203.

Pulfer, J. D and Whitehead, M. A., 2004. The hammerstein integral equation: a general technique for constructing a rapidly convergent padé-type approximation to the logarithmic derivative. International Journal of Quantum Chemistry, 8(5):693-706. DOI: 10.1002/qua.560080505.

Sakenov, D. Zh, etc., 2012. Preparation of students of higher education institution for professional activity in the course of studying of pedagogical disciplines. World applied sciences journal, 19 (10): 1431-1436.