WALIA journal 32(S1): 6-11, 2016
Available online at www.Waliaj.com
ISSN 1026-3861

Theoretical foundations quadratic transformation

Auyez Kenesbekuly Baidabekov [1, *], Baimuhan Nurmahanovich Nurmahanov [2], Atogali Alenovich Jumabaev [3], Abadikarim Almakhanovich Karazhanov [4], Elmira Abdenovna Kemelbekova [5]

[1](Professor, Doctor of Technical Sciences, Doctor PhD) L.N.Gumilyov Eurasian National University
[2](Professor, Doctor of Technical Sciences) L.N.Gumilyov Eurasian National University
[3](Doctor of Technical Sciences) L.N.Gumilyov Eurasian National University
[4](Candidate of engineering sciences) L.N.Gumilyov Eurasian National University
[5](Master) L.N.Gumilyov Eurasian National University

Abstract: The article is devoted to the set of new conformities four accordance's and on the design of canonical biquadratic transformations of plane and the method of receipt of biquadratic transformations of plane. This method allowed getting twelve types of canonical biquadratic transformations of plane, the worked out algorithm allowed to define the mathematical models of canonical biquadratic transformations of plane.

Key words: *Canonical Biquadratic; Transformations; Plane*

1. Introduction

From the analysis of scientific papers on applied geometry implies that the quadratic transformation plane adequately investigated and found application in science and technology. However, the study and application of the four - four-matches and biquadratic transformations of the plane is given little attention.

The method of applied geometry of quadratic transformation is effective in solving the positional problems associated with curves or surfaces; Using the quadratic transformation allows to halve the projected order of a curve or surface, which greatly simplifies the solution of a variety of positional problems (Cremona, 1863; Hudson, 1927; Skopets and Rosenfeld, 1952). It is not enough widespread use of quadratic transformations in applied geometry explained by the fact that the methods of quadratic transformations poorly developed, although the study of this problem, the development of graphical models and their application in applied geometry devoted dozens of leading experts in applied geometry.

Disambiguation point geometric transformations can be one - two-digit, three-digit one-one-four-digit, etc. Of these, one-double-digit point conversion investigated adequately.

One of the founders of Cremona (quadratic) transformation is Cremona L. (Cremona, 1863; Cremona, 1936). the initial information and are the foundations of the classical theory of non-linear birational (Cremona) transformations of the plane and three-dimensional space for the first time they were collected.

Hudson J. P (Hudson, 1927) in his monograph gave a more complete overview of the main issues of the classical theory of non-linear plane and three-dimensional space of birational transformations. Professor Skopets Z. A Skopets and Rosenfeld, 1952) The method of obtaining the special three-dimensional space of Cremona transformations was proposed. Research Skopets Z. A devoted to the study of the two - a two-digit conformity T_{2-2}.

2. The design concept of the establishment of the quadratic transformation between two planes aligned bones

Consider the spatial constructive scheme of establishing the four - three-valued correspondences between the two planes of the misalignment. Summary obtain four - four-matching misalignment between the planes Π_1 and Π_2 is as follows:

1. The three-dimensional Euclidean space E_3 set two intersecting surfaces of the second order algebraic Q_1^0 and Q_2^0. And set two planes generic projections Π_1 and Π_2 in accordance with Fig. 2.1.
2. Draw projecting beam S1, which crosses a given surface $Q_1^0$and Q_2^0, respectively, at the points A_1^0 and A_2^0, A_3^0 and A_4^0.
3. The surface of the second order Q_1^0 rotates around the x-axis OX_2 90^0 so that the positive direction of the axis OX_1 coincided with the negative OX_3 axis (Fig. 1).

We obtain a new position Q_1^{01} surface of the second order, and the point Q_1^0 A_1^{01} and A_2^{01}, which correspond to the points A_1^0 and A_2^0. Points A_1^{01} and A_2^{01} projecting projecting beam S_2 Π_2 on the plane, obtain points A_1 and A_2 (see Fig. 1).

[*] Corresponding Author.

4. OX_1 revolves around the axis of the second surface 90^0 of the second order Q_2^0 so that the positive direction of the axis OX_2 coincided with the negative OX_3 axis (Fig. 1). We obtain a new position Q_2^{01} surface of the second order, and the point Q_2^0 A_3^{01}, A_4^{01}, which correspond to the points A_3^0 and A_4^0. Projecting point A_3^{01} and A_4^{01} projecting beam S_2 Π_2 on the plane, we get the point A_3 and A_4.

5. Draw through the points A_1 and A_2 direct parallel axis OX_2. Also, we draw through the point A_3 and A_4 straight parallel OX_1 axis. These four straight lines intersect at points $A_1/$, $A_2/$ and $A_3/$, or $A_4/$ (Fig. 2).

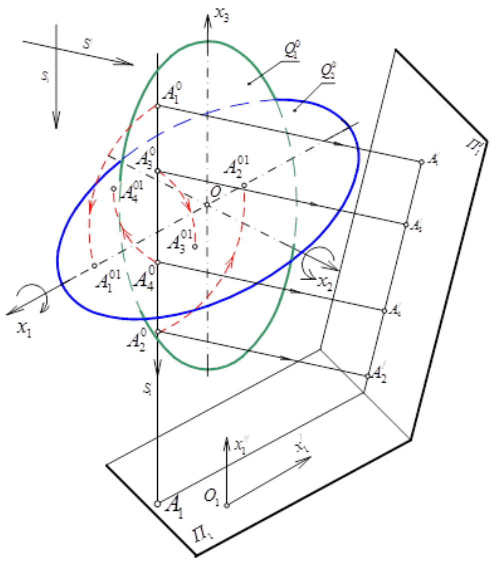

Fig. 1: Spatial design scheme

Thus, by sequentially performing the above structural unit A plane Π_1 at four points correspond to $A_1/$, $A_2/$, $A_3/$, $A_4/$ or Π_2 plane that is set four - four-line misalignment between the planes Π_1 and Π_2.

Now consider (4-4) -digit conversion between a combined planes, which is called by us biquadratic transformation plane.

The method of obtaining canonical quadratic transformations of the plane/

The essence of the proposed method of modeling quadratic transformations of the plane, binary image generated by the two surfaces of the second order, is as follows.

In Euclidean three-dimensional space E_3 defined the two surfaces of the second order Φ_1^0 and Φ_2^0, which have the form of the equation:

$$\Phi_1^0(x_1, x_2, x_3) = 0, \qquad (2.2.1)$$

$$\Phi_2^0(x_1, x_2, x_3) = 0, \qquad (2.2.2)$$

where x_1, x_2, x_3 - Cartesian coordinates;
Φ_1^0, Φ_2^0 – continuous second-order polynomial.

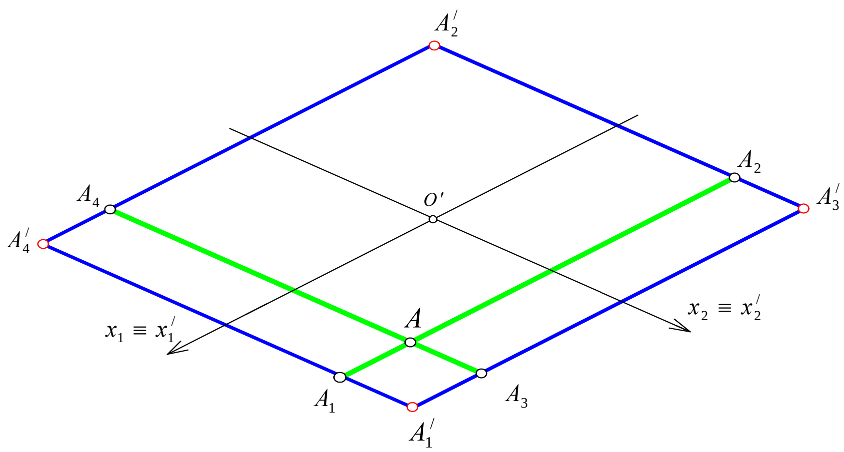

Fig. 2: Obtaining points $A_1/$, $A_2/$, $A_3/$, $A_4/$

Note on the plane Π_1 and the point A through this point will hold vertical beam S, which intersects the predetermined surface Φ_1^0 and Φ_2^0 respectively in the points A_1^0 and A_2^0, A_3^0 and A_4^0 (Fig. 2).

The surface of the second order of Φ_1^0 rotates around the axis of the ordinate so that the positive direction of the Z-axis coincides with the positive direction of the axis of abscissa.

In other words, second order surface Φ_1^0 is exposed three spatial transformation γ_1 (rotation around the axis of the ordinate at an angle of 90^0), which is given by the matrix equation:

$$\begin{pmatrix} X_1' \\ X_2' \\ X_3' \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} \qquad (2.2.3)$$

We obtain the new position of the second surface of the order of Φ_1^0 and point A_1^{01}, A_2^{01}, which correspond to the points A_1^0 and A_2^0. Point A_1^{01} and A_2^{01} projecting vertical beams on the plane Π_1, we get the point A_1 and A_2.

Revolves around the axis of the abscissae second surface Φ_2^0 of the second order so that the positive direction of the Z-axis coincides with the positive direction of the y-axis.

Thus, the surface spatial transformation subjecting Φ_2^0 or γ_2 specified by the matrix equation:

$$\begin{pmatrix} X_1' \\ X_2' \\ X_3' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} \quad (2.2.4)$$

After conversion we obtain a new position of the surface Φ_2^0 and the second-order points A_3^{01} and A_4^{01}, which correspond to the points A_3^0 and A_4^0. The projecting points A_3^{01} and A_4^{01} on the vertical plane of the beams Π_1, A_3 and get A_4 points.

After point A_1, A_2 and A_3, A_4 hold the straight lines parallel to the coordinate axes, respectively, *OX₂, OX₁*. We obtain a quadrilateral with vertices $A_1^/$, $A_2^/$ and $A_3^/$, $A_4^/$.

Sequential performance of the above mentioned structural unit, each point *A* plane Π_1 is converted to four dots $A_1^/$, $A_2^/$ and $A_3^/$, $A_4^/$ of the plane $\Pi_1^/$.

Given the two-parameter set points combined plane $\Pi_1^/ \equiv \Pi_1$ we obtain the quadratic transformation of the plane, marked with a L. Similarly, we can show that in the opposite direction of each point of the plane $A^/$ or $\Pi_1^/$ is converted into four points Π_1 plane. This transformation is denoted by the letter $L^/$.

Foregoing allows us to form the following theorem:

Theorem: If you specify two rotation surfaces Φ_1^0 and Φ_2^0, respectively, which are subject to spatial transformations, γ_1 and γ_2, and they are displayed on the lines S and $s^/$ on the plane $\Pi_1^/ \equiv \Pi_1$, is set biquadratic transformation *L* and *L/ or* $\Pi_1^/ \equiv \Pi_1$ between the combined planes.

With the use of the above-proposed space constructive schemes will get different types of canonical transformations biquadratic, L, *L/* plane, covered in the next section.

3. Development of canonical quadratic transformations of the plane

In order to obtain a quadratic transformation binary plane displayed on the two surfaces of the second-order plane. At the same time we consider three cases: a) a combination of nonruled surfaces of the second order; b) a combination of cylindrical and conical surfaces of the second order; c) a combination of hyperboloid of second order. As a result of these cases received three subgroups of the plane transformation biquadratic.

4. Quadratic transformations of the plane generated by a binary display of the two ruled surfaces 2nd order

To simulate the first subgroup of quadratic transformations of the plane, in the above spatial design scheme considering the case where the combination of binary mapping of the surface of the second order is non-ruled surfaces 2nd order, such as a sphere and two-sheeted hyperboloid.

Define a list of second-order ruled surfaces such as a sphere, hyperboloid of one sheet in the x, y, and the z, serving on the twin elements of spatial constructive scheme. Of these four surfaces created twelve combinations of options displayed surfaces Φ_1^0 and Φ_2^0.

Studies have shown that the degenerate form (4-4) -valued fit plane, and two-way hyperboloid along the x axis, and the z, yielded four types of biquadratic canonical transformations of the plane *L* and *L/*.

Consider the example of a simulation of a quadratic transformation, when the first surface Φ_1^0 is a two-sheeted hyperboloid with the real axis *OX₃* and the second surface Φ_2^0 is a two-sheeted hyperboloid with the real axis *OX₁.*

According to the method proposed in section 2.2, the surface Φ_1^0 is exposed to transform

$$\begin{pmatrix} X_1' \\ X_2' \\ X_3' \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} \quad (2.3.1)$$

and then displayed on the orthogonal projection plane Π_1.

The surface Φ_2^0 undergoes transformation

$$\begin{pmatrix} X_1' \\ X_2' \\ X_3' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} \quad (2.3.2)$$

and then displayed on the orthogonal projection plane $\Pi_1/$.

As a result of the above method, each point of the plane Π_1 is converted into a four-point plane $\Pi_1^/$. Taking into account the two - parameter set points $\Pi_1^/ \equiv \Pi_1$ combined the plane, we obtain biquadratic transformation of the plane. Similarly, we can show that in the reverse direction, each point of the plane $\Pi_1^/$ is converted into Π_1 four-point plane.

5. Quadratic transformations of the plane, binary image generated by the conical and cylindrical surfaces of the second order

To simulate the second subgroup of quadratic transformations of the plane consider the case where a combination of binary mapping of the surface of the second order is ruled surfaces 2nd order, such as conical and cylindrical surface of revolution.

Because these surfaces are created thirteen combinations of options displayed surfaces Φ_1^0 and Φ_2^0.

Studies have shown that items with serial numbers from 2 to 4 th, 6 th and 9 to 13, form a degenerate (4-4) -valued fit plane, and items with serial numbers 1, 5, 7 and 8 allowed get four kinds of biquadratic canonical transformations *L* and *L/*.

Consider the example of modeling a quadratic transformation plane when the first surface Φ_1^0 a round cone OX_1 the real axis and the second surface Φ_2^0 is circular cone with OX_3 real axis.

According to the method proposed in Section 2.2, second order surface Φ_1^0 OX_1 revolves around an axis so that the positive direction OX_3 axis coincides with the positive direction of the axis OX_1. In other words, the tapered surface of the second order Φ_2^0 undergoes rotational transformation around the axis OX_1

$$\begin{pmatrix} X_1' \\ X_2' \\ X_3' \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix}, \qquad (2.3.3)$$

and then displayed on the orthogonal projection plane Π_1.

Conical surface of the second order of Φ_2^0 revolves around the axis OX_1 so that the positive direction OX_3 axis coincides with the positive direction of the axis OX_2. In other words, the surface Φ_2^0 subjected to rotation transformation to 90^0.

$$\begin{pmatrix} X_1' \\ X_2' \\ X_3' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix}, \qquad (2.3.4)$$

and then displayed on the orthogonal projection plane $\Pi_1/$.

As a result of consistent implementation of the above method, each point Π_1 plane is converted into four points of the plane $\Pi_1^/$. Considering the two-parameter set points combined plane $\Pi_1^/ \equiv \Pi_1$, we obtain biquadratic transformation of the plane. Similarly, we can show that in the reverse direction, each point of the plane $\Pi_1^/$ is converted into $\Pi_1/$ four-point plane.

6. Quadratic transformations of the plane, binary image generated by the two ruled hyperbolic surfaces 2nd order

To simulate the third subgroup of quadratic transformations of the plane, consider the case where the combination of second-order binary mapping of the surface is non-ruled surfaces 2nd order, such as a sphere and two-sheeted hyperboloid.

Of these seven surfaces created twenty-two combinations of options displayed surfaces Φ_1^0 and Φ_2^0.

Studies have shown that the sequence numbers 3, 4, 6, 7, 8, 10 and 11 th to 22 th degenerate form (4-4) -valued fit plane, and with sequence numbers 1, 2, 5 and 9 possible to obtain four kinds of biquadratic canonical transformations of the plane *L* and *L/*.

Consider the example of a simulation of a quadratic transformation, when the first surface Φ_1^0 is a one-sheeted hyperboloid with the real axis OX_3 and the second surface Φ_2^0 is a one-sheeted hyperboloid with OX_1 axis.

According to the method proposed in section 2.2, the surface is subjected to rotational transformation Φ_1^0 or 90^0.

$$\begin{pmatrix} X_1' \\ X_2' \\ X_3' \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix}. \qquad (2.3.7)$$

and then displayed on the orthogonal projection plane $\Pi_1/$.

The surface of the second order of Φ_2^0 revolves around the axis OX_1 so that the positive direction OX_3 axis coincides with the positive direction of the axis OX_2. That is, the surface Φ_2^0 is being transformed, given the matrix equation (2.3.8) and then display the orthogonal projection on the plane $\Pi_1/$.

$$\begin{pmatrix} X_1' \\ X_2' \\ X_3' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} \qquad (2.3.8)$$

Sequential performance of the above structural unit, each point of the plane Π_1 is converted into a four-point plane $\Pi_1^/$. Thus, the parameter set in two points of the plane $\Pi_1^/ \equiv \Pi_1$ biquadratic obtain transform plane *L*. Similarly, we can show that in the reverse direction, each dot $A^/$ is converted to plane $\Pi_1^/$ four points in the plane Π_1, it will be inverse *L/*.

7. Definitions equation quadratic canonical transformations

Biquadratic transformation plane are mutually (1-4) - one correspondence between the points of two planes Π_1 and $\Pi_1^/$.

Equations direct conversion biquadratic *L* are determined by the following algorithm. The equation

$$\Phi_1^0(x_1, x_2, x_3) = 0, \qquad (2.2.1)$$

Φ_1^0 surface rewritten as

$$x_3 = \varphi_1(x_1, x_2) \qquad (2.4.1)$$

The surface of the second order of Φ_1^0 undergoes rotation transformation around the x-axis at an angle of 90^0. According to the equation (2.2.3), we obtain:

$$x_1^{/} = x_3. \qquad (2.4.2)$$

In view of the equation (2.4.1), from (2.4.2) we get:

$$x_1^{/} = \varphi_1(x_1, x_2) \qquad (2.4.3)$$

Equations Φ_2^0 surface

$$\Phi_2^0(x_1, x_2, x_3) = 0, \qquad (2.2.2)$$

rewrite as

$$x_3 = \varphi_2(x_1, x_2). \qquad (2.4.4)$$

The surface of the second order of Φ_2^0 undergoes rotation transformation around the ordinate axis 90^0. According to equation (2.2.4), we obtain:

$$x_2^{/} = \varphi_2(x_1, x_2) \qquad (2.4.5)$$

By combining in a single system of (2.4.3) and (2.4.5), we obtain the desired formula biquadratic direct conversion L:

$$\begin{cases} x_1^{/} = \varphi_1(x_1, x_2), \\ x_2^{/} = \varphi_2(x_1, x_2). \end{cases} \qquad (2.4.6)$$

Using the proposed algorithm below received canonical equation quadratic transformations of the plane L in the corresponding Table 1.

Table 1: Mathematical models of plane transformations biquadratic

Designation and the equation *L*	Designation of the equation and L^1changes
$L_1: \begin{cases} x_1' = \sqrt{x_1^2 + x_2^2 + R^2} \\ x_2' = \sqrt{x_1^2 - x_2^2 - R^2} \end{cases}$	$L_1': \begin{cases} x_1 = \sqrt{\frac{x_1'^2 + x_2'^2}{2}} \\ x_2 = \sqrt{\frac{x_1'^2 - x_2'^2 - 2R^2}{2}} \end{cases}$
$L_2: \begin{cases} x_1' = \sqrt{x_1^2 + x_2^2 + R^2} \\ x_2^1 = \sqrt{x_2^2 - x_1^2 - R^2} \end{cases}$	$L_2': \begin{cases} x_1 = \sqrt{\frac{x_1'^2}{2} - \frac{x_2'^2}{2}} \\ x_2 = \sqrt{\frac{x_1'^2 + x_2'^2}{2}} \end{cases}$
$L_3: \begin{cases} x_1' = \sqrt{x_1^2 - x_2^2 - R^2} \\ x_2^1 = \sqrt{x_1^2 + x_2^2 + R^2} \end{cases}$	$L_3': \begin{cases} x_1 = \sqrt{\frac{x_1'^2}{2} + \frac{x_2'^2}{2}} \\ x_2 = \sqrt{\frac{x_2'^2 - 2R^2 - x_1'^2}{2}} \end{cases}$
$L_4: \begin{cases} x_1' = \sqrt{x_2^2 - x_1^2 - R^2} \\ x_2^1 = \sqrt{x_1^2 + x_2^2 + R^2} \end{cases}$	$L_4': \begin{cases} x_1 = \sqrt{\frac{x_2'^2}{2} - \frac{x_1'^2}{2} - R^2} \\ x_2 = \sqrt{\frac{x_2'^2 - x_1'^2}{2}} \end{cases}$
$L_5: \begin{cases} x_1' = \sqrt{x_1^2 - x_2^2} \\ x_2^1 = \sqrt{x_1^2 + x_2^2} \end{cases}$	$L_5': \begin{cases} x_1 = \sqrt{\frac{x_1'^2 + x_2'^2}{2}} \\ x_2 = \sqrt{\frac{x_2'^2 - x_1'^2}{2}} \end{cases}$
$L_6: \begin{cases} x_1' = \sqrt{x_2^2 - x_1^2} \\ x_2^1 = \sqrt{x_1^2 + x_2^2} \end{cases}$	$L_6': \begin{cases} x_1 = \sqrt{\frac{x_2'^2 - x_1'^2}{2}} \\ x_2 = \sqrt{\frac{x_2'^2 + x_1'^2}{2}} \end{cases}$
$L_7: \begin{cases} x_1' = \sqrt{x_1^2 + x_2^2} \\ x_2' = \sqrt{x_1^2 - x_2^2} \end{cases}$	$L_7': \begin{cases} x_1 = \sqrt{\frac{x_1'^2 + x_2'^2}{2}} \\ x_2 = \sqrt{\frac{x_1'^2 - x_2'^2}{2}} \end{cases}$
$L_8: \begin{cases} x_1' = \sqrt{x_1^2 + x_2^2} \\ x_2' = \sqrt{x_2^2 - x_1^2} \end{cases}$	$L_8': \begin{cases} x_1 = \sqrt{\frac{x_1'^2 - x_2'^2}{2}} \\ x_2 = \sqrt{\frac{x_1'^2 + x_2'^2}{2}} \end{cases}$
$L_9: \begin{cases} x_1' = \sqrt{x_1^2 + x_2^2 - R^2} \\ x_2' = \sqrt{x_1^2 - x_2^2 + R^2} \end{cases}$	$L_9': \begin{cases} x_1 = \sqrt{\frac{x_1'^2}{2} + \frac{x_2'^2}{2}} \\ x_2 = \sqrt{\frac{x_1'^2}{2} - \frac{x_2'^2}{2} + R^2} \end{cases}$
$L_{10}: \begin{cases} x_1' = \sqrt{x_1^2 + x_2^2 - R^2} \\ x_2' = \sqrt{x_2^2 - x_1^2 + R^2} \end{cases}$	$L_{10}': \begin{cases} x_1 = \sqrt{\frac{x_1'^2}{2} - \frac{x_2'^2}{2} + R^2} \\ x_2 = \sqrt{\frac{x_1'^2 + x_2'^2}{2}} \end{cases}$
$L_{11}: \begin{cases} x_1' = \sqrt{x_1^2 - x_2^2 + R^2} \\ x_2' = \sqrt{x_1^2 + x_2^2 - R^2} \end{cases}$	$L_{11}': \begin{cases} x_1 = \sqrt{\frac{x_1'^2}{2} + \frac{x_2'^2}{2}} \\ x_2 = \sqrt{\frac{x_2'^2}{2} - \frac{x_1'^2}{2} + R^2} \end{cases}$
$L_{12}: \begin{cases} x_1' = \sqrt{x_2^2 - x_1^2 + R^2} \\ x_2' = \sqrt{x_1^2 + x_2^2 - R^2} \end{cases}$	$L_{12}': \begin{cases} x_1 = \sqrt{\frac{x_2'^2}{2} - \frac{x_1'^2}{2} + R^2} \\ x_2 = \sqrt{\frac{x_2'^2 + x_1'^2}{2}} \end{cases}$

8. Conclusions

1. The developed spatial construction diagram display of two second-order surfaces allowed to establish new patterns of producing four-four-correspondences between the two planes of the misalignment.

2. Create the theoretical principles modeling biquadratic canonical transformations of the plane, and developed a method for biquadratic transformation plane porazhdaet two second order surface mapping binary on a combined plane. This method allowed obtaining twelve kinds of biquadratic canonical transformations of the plane.

3. The developed algorithm has allowed defining mathematical models biquadratic canonical transformations of the plane.

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