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Vibration control using tuned mass damper

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Abstract: Civil Engineering structures have to withstand natural environmental forces like wind, earthquake forces and wave forces, along with loads that they are designed to resist. All this environmental forces are random and dynamic in nature. Therefore the response of the structure is also dynamic and that is what causes the unsafe and uncomfortable conditions. Therefore there is always a need for some sort of control of response of structure. This project aims at studying both methods of the Tuned Mass Dampers. It has been well established that Single tuned mass damper (STMD) and multiple tuned mass damper (MTMD) are effective in reducing the response of the structure. The project aims and study of two devices, the Single Tuned Mass Damper and Multiple Tuned Mass Damper using new control strategy. The tuned mass dampers, consisting of one larger mass block (i.e. one larger tuned mass damper) and one smaller mass block (i.e. one smaller tuned mass damper), referred in this report as the STMD, have been studied to seek for the mass dampers with high effectiveness and robustness for the reduction of the undesirable vibrations of structures under the ground acceleration. Multiple tuned mass dampers (MTMD) consisting of many active tuned mass dampers (TMDs) with uniform distribution of natural frequencies have been, proposed to attenuate undesirable oscillations of structures under the ground acceleration.

Key words: *Vibration; Damper; Tuned mass damper (Single and multiple)*

1. Introduction

Civil Engineering structures have to withstand environmental forces like wind, earthquake forces and wave forces along with loads that they are designed to resist. All this environmental forces are random and dynamic in nature. Therefore the response of the structure is also dynamic and that is what causes the unsafe and uncomfortable conditions. Therefore there is always a need for some sort of control of response of structure. The fact is more important in present times due to following factors:

1. Increased flexibility: it is now a necessity and trend to use tall, long or in general more flexible structures. There is also a growing tendency to use lighter and more flexible construction materials. These factors promote the idea of control of vibrations of structure.
2. Increased safety levels: As structure becomes more complex, costly and as it serves more critical function, it demands higher safety levels.
3. Stringent performance requirements: Structures are required to respond to the forces acting on them within the safety limits. Hence for environmental loads, which are random and dynamic in nature, more stringent safety limits are generally set, which demand for control of vibrations of the structure. Due to the above listed reasons, the concept of structural perception using control systems is not only becoming increasingly popular but it is becoming almost a necessity in modern days.

The Tuned Mass Damper is a classical engineering device that is used for vibration control. It consists of mass, a spring and a damper, which is attached to the main structure Fig. 1. Single tuned mass

Dampers (STMD) have proved to be very sensitive even to the small offset in tuning ratio when it is optimally designed. This is the greatest disadvantage of STMD. This is due to following reasons. Errors in predicting or identifying the natural frequency of the structure and also the error in fabricating a TMD are inevitable to some degree. Some structures have nonlinear properties even in small amplitude range due to contribution of secondary members.

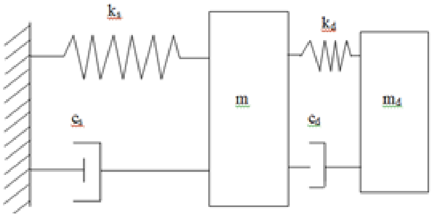

Fig. 1: The Single tuned mass damper

Therefore, in practical design the optimum values of parameters of TMD are not chosen. The damping of the TMD is intentionally made higher than the optimal value such that TMD become less sensitive to tuning errors. This results increase the mass of TMD to meet the design requirement.

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All these uncertainties can be reduced by use of Multiple Tuned Mass Dampers (MTMD). Use of MTMD has been proposed to increase the robustness of the vibration control system to various uncertainties in the structures and/or TMD.

The basic configuration of MTMD is the large number of small oscillators whose natural frequencies are distributed around the natural frequency of the controlled mode of structure. It is now well established that an optimal MTMD is more effective and robust than optimal STMD.

This project aims at studying both methods of Tuned Mass Dampers. It has been well established that STMD and MTMD are effective in reducing the response of the structure.

The main objective of the present study is to devise a mass distribution scheme to attain the following goals.

1. Reduce the maximum value of Dynamic Magnification Factor (DMF) by trying different types of mass distributions.

2. Attaining a perfectly flat DMF curve over a significant bandwidth of frequency ratios.

2. Literature review

Tuned mass dampers (TMD) are widely used to control the vibrations in civil engineering structures. Although TMDs are effective in reducing the vibrations caused by stationary excitation forces, their performance to suppress seismic response is limited. This inefficiency is due to the fact that TMDs usually need some time interval before it becomes fully effective because they are initially at rest, while the strongest seismic ground motion is often observed at the earlier stage of an earthquake. Another drawback is that TMDs are sensitive to tuning error. Employing more than one tuned mass damper with different dynamic characteristics has then been proposed to further improve the effectiveness and robustness of the TMD. The multiple tuned mass dampers (MTMD) with the distributed natural frequencies were proposed by Xu and Igusa [1991] and also studied by, Abe and Fujino [1994], Abe and Igusa [1995], Bakre and Jangid [2004], Chen and Wu [2001], The MTMD is shown to possess better effectiveness and higher robustness in mitigating the oscillations of structures with respect to a single TMD.

Likewise, the dual-layer multiple tuned mass dampers, referred to as the DL-MTMD consisting of one larger tuned mass damper and several smaller tuned mass dampers with the total number of tuned mass damper units being the arbitrary integer and with the uniform distribution of natural frequencies have been further proposed by Li [2005] to seek for the mass dampers with high effectiveness and robustness for the reduction of the undesirable vibrations of structures under the ground acceleration. The numerical results indicate that the DL-MTMD can render better effectiveness and higher robustness to the change in the natural frequency tuning (NFT), in comparison with the multiple tuned mass dampers (MTMD) with equal total mass. In fact the DL-MTMD will degenerate into the double tuned mass damper when the total number of the smaller tuned mass damper units in the DL-MTMD is set to be equal to unity. The investigations by Li [2005] have manifested that the DL-MTMD has a little better effectiveness with respect to the DTMD, but they practically reach the same level of robustness to the change in the natural frequency tuning (NFT). The DTMD consists of one larger mass block (larger tuned mass damper) and one smaller mass block (i.e. smaller tuned mass damper), thus implying that it is significantly simpler to manufacture the DTMD in comparison with the DL-MTMD. With a view to the engineering design and practical applications, it is imperative and of practical interest to carry on further investigations on the DTMD.

Active TMDs can be effective in reducing seismic response because the TMD amplitude can be increased much faster through the use of the actuators. They can also be more robust to tuning errors with the appropriate use of feedback. Therefore, active TMDs have attracted broad research interest and various control algorithms have been developed Yang et al. [1987]; Spencer et al. [1994]; Chang and Yang (1995). Because of their efficiency and compactness, active TMDs have been successfully designed and installed in full scale Kobori [1991].

Yao [1972] made an attempt to stimulate interest among structural engineers in the application of control theory in the design of civil engineering structures. This has been concluded that much more work is needed in order to apply the concept of structural control to complicated structures such as extremely tall buildings or long bridges subjected to uncertain dynamic loads such as wind and earthquake excitations.

Modern control theories that were developed during the past decade have been successfully applied to the control of the trajectory and motions of space vehicles as well as aeronautical systems. Recently, the control theory has also been applied to reduce the vibration of civil engineering structures Yang [1975]. The major difficulty to be encountered is that most civil engineering structures have been very heavy.

Experiments on active control of Seismic Structures have been presented by Chung et al. [1998] in which the first phase of a comprehensive experimental study concerning the possible application of active control to structures under seismic excitations is discussed. The experiment consisted of a single degree of freedom model structure, controlled using prestressing tendons connected to the servo hydraulic actuators. An optimal closed loop control scheme using a quadratic performance index was employed to reduce the response of structure under base motion generated by a large scale seismic simulator. Using a carefully designed, fabricated, and calibrated experimental setup the correlation between the analytical and experimental results was studied. Based on similitude relations, the experimental results obtained for the model structure was extrapolated to the full scale structures are analyzed.

Reinhom et al. [1987] presented a methodology for the shape control of structures undergoing inelastic deformations through the use of an active pulse force system. To avoid the large deformation in structures like tall buildings, long bridges and offshore platforms external forces are applied to the structure through cables, air jets, or other devices in order to ensure that the deformations are kept below the limits set for serviceability at all times.

Yang [1975] investigated the feasibility of optimum active control theory for controlling the motion and vibration of civil engineering structures. It is assumed that the structural system can be discredited, such that the equation of motion can be described by a system of ordinary differential equations. The effectiveness of the control system is measured by a performance index. The optimal control law, which minimizes the performance index, is a linear feedback control.

The optimal control forces are obtained by solving a matrix Ricatti equation. Moreover, the feasibility of implementing the active control by means of active dampers and servomechanism is considered there.

Abe [1996] is also proposing a rule based on control algorithm for active TMDs. First, perturbation solutions of the linear quadratic regulator (LQR) feedback gains for the active TMD system are derived. Using these solutions interaction of the TMD and the actuator force is discussed in detail. The algorithm consisted of two parts: (1) a variable gain displacement feedback control (2) a variable TMD damping control. The first one is applied when the TMD amplitude is small to make the TMD more effective, and the second one is applied when the TMD amplitude is large to dissipate the energy.

Sarbjeet et al. [1998] presented a control strategy based on the combination of feed forward and feedback gain controls (an open-closed loop) for the reduction of the displacement response of the shear frame model of tall buildings to random ground motion which is represented by double filtered white noise.

3. Methodology

3.1. Single tuned mass damper

Tuned mass damper (TMD) is a widely used passive energy absorbing device consisting of a secondary mass, a spring, and a viscous damper. It is attached to a primary or main vibratory system to reduce its dynamic motion. TMD was first suggested by Frahm in 1909. So its effectiveness depended on the closeness of absorber's natural frequency to the excitation frequency. First closed form expressions for optimum parameters of a TMD were derived by Den Hartog 44 for an undamped single degree of freedom (SDOF) main structure subjected to harmonic force. Since then, optimum parameters of TMD have been studied extensively. TMD is usually designed by modeling the main structure as an equivalent SDOF structure 2.

3.2. Optimum parameters for STMD

Fig. 2 shows a single degree of freedom (SDOF) system having mass and stiffness subjected to both external forcing and ground motion. A tuned mass damper with mass md and stiffness kd is attached to the primary mass. The various displacement measures are: ug the absolute ground motion; u, the relative motion between the primary mass and the ground; and ud the relative between the damper and the primary mass.

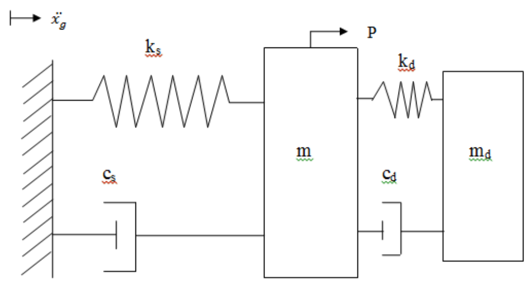

Fig. 2: Undamped SDOF system coupled with damped TMD system

The equations of motion for this case are:

$$m_d\ddot{u}_d + c_d\,\dot{u}_d + k_d u_d + m_d\ddot{u} = -m_d a_g \quad (3.1)$$

$$m\ddot{u} - c_d\,\dot{u}_d - k_d u_d + ku = -m a_g + P \quad (3.2)$$

The inclusion of the damping terms in eqns (3.1) and (3.2) produces a phase shift between the periodic excitation and the response. It is convenient to work initially with the solution expressed in terms of complex quantities. One expresses the excitation as

$$a_g = \hat{a}_g \varepsilon^{ipt} \quad (3.3)$$

$$P = \hat{p}\varepsilon^{ipt} \quad (3.4)$$

Where $\hat{a}_g$ and $\hat{p}$ are real quantities. The response is taken as

$$u = \bar{u}\varepsilon^{ipt} \quad (3.5)$$

$$u_d = \overline{u_d}\varepsilon^{ipt} \quad (3.6)$$

Where the response amplitudes, $\bar{u}$ and $\overline{u_d}$ are considered to be complex quantities. The real and imaginary parts of a_g correspond to cosine and sinusoidal input. Then, the corresponding solution is given by either the real (for cosine) or imaginary (for sine) parts of u and u_d. Substituting eqns (3.5) and (3.6) in the set of governing equations and cancelling from both sides results in

$$[-m_d p^2 + ic_d + k_d]\overline{u_d} - m_d p^2 \bar{u} = -m_d \hat{a}_g \quad (3.7)$$

$$-[ic_d p + k_d]\overline{u_d} + [-m p^2 + k]\bar{u} = -m\hat{a}_g + \hat{p} \quad (3.8)$$

The solution of the governing equation is,

$$\bar{u} = \frac{\hat{p}}{kD_2}[\gamma^2 - \rho^2 + 12\xi_d \rho\gamma] - \frac{\hat{a}_g m}{kD_2}[(1+\mu)\gamma^2 - \rho^2 + 12\xi_d \rho\gamma(1+\mu)] \quad (3.9)$$

$$\bar{u}_d = \frac{\hat{p}}{kD_2}\rho^2 - \frac{\hat{a}_g}{kD_2} m \quad (3.10)$$

Where

$$D_2 = [(1-\rho^2)(\gamma^2 - \rho^2) - \mu\rho^2\gamma^2 + 12\xi_d \rho\gamma(1-\rho^2(1+\mu))] \quad (3.11)$$

$$\gamma = \frac{\omega_d}{\omega_s} \quad (3.12)$$

and ρ is the frequency ratio. Converting complex solution to polar form leads to the following expression,

$$\bar{u} = \frac{\hat{p}}{k} H_1 e^{i\delta_1} - \frac{\hat{a}_g}{k} m H_2 e^{i\delta_2} \quad (3.13)$$

$$\bar{u}_d = \frac{\hat{p}}{k} H_3 e^{i\delta_3} - \frac{\hat{a}_g}{k} m H_4 e^{i\delta_3} \quad (3.14)$$

where the H factors define the amplification of the pseudo-static responses, and the d's are the phase angles between the response and the excitation. The various H and d terms are listed below.

$$H_1 = \frac{\sqrt{[\gamma^2 - \rho^2] + [2\xi_d \rho\gamma]^2}}{|D_2|} \quad (3.15)$$

$$H_2 = \frac{\sqrt{[(1+\mu)\gamma^2 - \rho^2]^2 + [2\xi_d \rho\gamma(1+\mu)]^2}}{|D_2|} \quad (3.16)$$

$$H_3 = \frac{\rho^2}{|D_2|} \quad (3.17)$$

$$H_4 = \frac{1}{|D_2|} \quad (3.18)$$

$$|D_2| = \sqrt{([1-\rho^2 I\gamma^2 - \rho^2] - \mu\rho^2\gamma^2)^2 + (12\xi_d)\rho\gamma[1-\rho^2(1+\mu)]^2}$$
(3.19)

Also δ's which is phase angles between response and excitation are given as,

$$\delta_1 = \alpha_1 - \delta_3 \quad (3.20)$$

$$\delta_2 = \alpha_2 - \delta_3 \quad (3.21)$$

$$\tan\delta_3 = \frac{2\xi_d \rho\gamma[1-\rho^2(1+\mu)]}{[1-\rho^2 I\gamma^2 - \rho^2] - \mu\rho^2\gamma^2} \quad (3.22)$$

$$\tan\alpha_1 = \frac{2\xi_d \gamma}{\gamma^2 - \rho^2} \quad (3.23)$$

$$\tan\alpha_2 = \frac{2\xi_d \rho\gamma(1+\mu)}{(1+\mu)\gamma^2 - \rho^2} \quad (3.24)$$

For most applications, the mass ratio is less than about 0.05. Then, the amplification factors for external loading (H_1) and ground motion (H_2) are essentially equal. A similar conclusion applies for the phase shift. In what follows, the solution corresponding to ground motion is examined and the optimal values of the damper properties for this loading condition are established. An in-depth treatment of the external forcing case is contained in Den Hartog's text. Fig. 3 shows the variation of H_2 with forcing frequency for specific values of damper mass μ and γ (as defined in eq 3.13), and various values of the damper damping ratio, ξ_d.

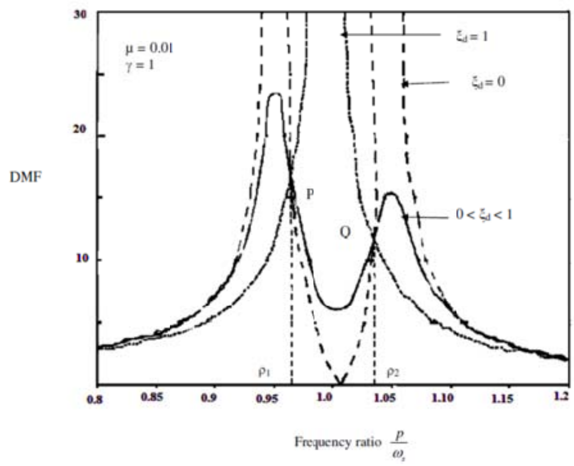

Fig. 3: Amplitude of main mass for various values of absorber damping

The family of DMF curves passes through two fixed points. Each curve represents the response factor for a particular damping ratio. When ξ_d=0, there are two peaks with infinite amplitude located on each side of frequency ratio, ρ=1. The behavior of the amplitudes suggests that there is an optimal value of ξ_d for a given damper configuration. Another key observation is that all the curves pass through two common points, P and Q. Since these curves correspond to different values of ξ_d, the location of P and Q must depend only on μ and γ. Using this property, Den Hartog [1956] evaluated the frequency such that two fixed points represents equal amplitudes. The damping was then modified to make the response curve pass through both the points with a horizontal tangent at each point. This yielded two values of damping, the average of which was taken to be the optimum.

ρ_1 and ρ_2 are the frequency ratios corresponding to points P and Q.

Proceeding with the above line of reasoning H_2 can be written as,

$$H_2 = \sqrt{\frac{a_1^2 + \xi_d^2 a_2^2}{a_3^2 + \xi_d^2 a_4^2}} = \frac{a_2}{a_4}\sqrt{\frac{a_1^2 / a_2^2 + \xi_d^2}{a_3^2 / a_4^2 + \xi_d^2}} \quad (3.25)$$

where the '*a*' terms are functions of μ, ρ and f. Then, for H_2 to be independent of ξ_d, the following condition must be satisfied.

$$\left|\frac{a_1}{a_2}\right| = \left|\frac{a_3}{a_4}\right| \tag{3.26}$$

Substituting for the 'a' terms, one obtain a quadratic equation for ρ^2

$$\rho^4 - \left[(1+\mu) f^2 + \frac{1+0.5\mu}{1+\mu}\right]\rho^2 + f^2 = 0 \tag{3.27}$$

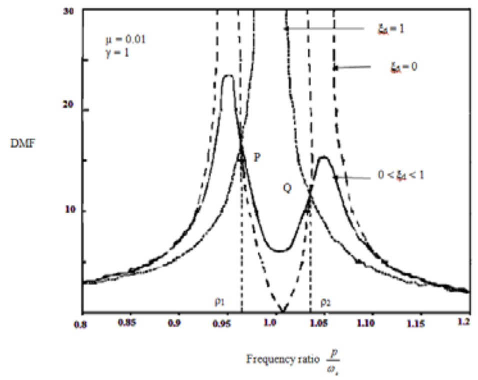

Fig. 2: Amplitude of main mass for various values of absorber damping.

The two positive roots ρ_1 and ρ_2 are frequency ratios corresponding to P and Q.

As a first step, one requires the values of for ρ_1 and ρ_2 to be equal. This produces a distribution which is symmetrical about $\rho^2 = \frac{1}{1+\mu}$, as illustrated in Fig. 4. Then, by increasing the damping ratio, ξ_d one can lower the peak amplitudes until the peaks coincide with points P and Q. This state represents the optimal performance of the TMD system. A further increase in ξ_d causes the peaks to merge and the amplitude to increase beyond the optimal value.

Requiring amplitudes to be equal at P and Q is equivalent to the following condition on root,

$$\left|1 - \rho_1^2 (1+\mu)\right| = \left|1 - \rho_2^2 (1+\mu)\right| \tag{3.28}$$

The optimum values thus obtained are as follows,

$$f_{opt} = \sqrt{1 - 0.5\mu}\,\frac{1}{1+\mu} \tag{3.29}$$

3.3. Conclusions

The family of DMF curves passes through two fixed points and each curve represents the response factor for a particular damping ratio. The location of P and Q must depend only on μ and γ. Dan Hartog show the phase shift from two curves to one curve.

$$\rho_{1,2}\big|_{opt} = \sqrt{\frac{1 \pm \sqrt{0.5\mu}}{1+\mu}} \tag{3.30}$$

$$\xi_d\big|_{opt} = \sqrt{\frac{\mu\left(3 - \sqrt{0.5\mu}\right)}{8(1+\mu)(1-0.5\mu)}} \tag{3.31}$$

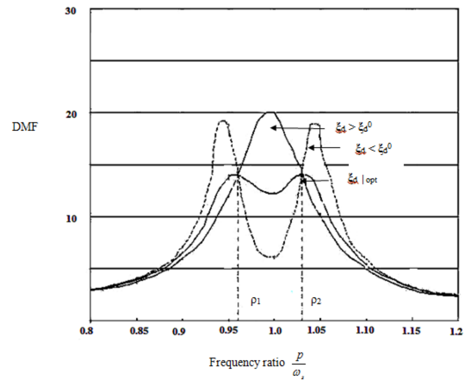

Fig. 3: Response curve for amplitude of system with optimally tuned TMD

3.4. Multiple tuned mass damper:

In the 90's, studies on the application of multiple tuned mass dampers for one-degree of freedom systems were started Xu and Igusa [1991]. It has been proved that MTMD with distributed natural frequencies are more effective than single TMD. The studies of MTMD were also developed in Kareem and Klime [1995], Jangid [1995]. Later on, structures subjected to seismic loads, treated as a multi degree of freedom structures and with the MTMD on them were analyzed in Chen and Wu [2001]. The MTMD were designed in such a way that they are tuned to several modes of structure vibration. The number of dampers depends on the number of vibration modes for which dampers are tuned. The performance of multiple mass dampers under both wind and seismic excitation is analyzed by Kareem and Kline [1995].

The effectiveness and robustness of a particular version of MTMD called the "multiple dual tuned mass dampers", is analyzed in the paper Han and Li [2005]. The problem of determination of optimum properties of MTMD is considered in the papers Li and Qu [2006], Li [2000].

Several studies on the effectiveness of the MTMD are presented. The optimum parameters of the MTMD system for a base-excited main system are presented. The criterion selected for optimality is minimization of the r.m.s. displacement of the main system. The base excitation is modelled as a white noise stationary random process. The optimum parameters of the MTMD system are obtained for different damping ratios of the main system and mass ratios of the MTMD. This may find application in the effective design of MTMD for base-excited systems. Furthermore the optimum parameters of the MTMD system are compared with those of a corresponding single tuned mass damper system9.

The basic configuration of a multiple tuned mass damper comprises of number of TMD's attached to the main structure as shown in Fig.5. The system configuration consists of a main system supported by n tuned mass dampers with different dynamic characteristics as shown below. The main system is characterized by natural frequency ω_s damping ratio ξ_s and mass m_s. The main system and each TMD are modelled as a single degree of freedom system so that the total degree of freedom of the structural system is n+1.

The basic assumptions made here are

1. Structure

• The natural frequencies of the structure are not closely spaced.

• The vibration to be suppressed is in specific vibration mode.

2. MTMD's

• The damping ratio of each TMD is same.

• The natural frequencies of the MTMD's are equally distributed (equal natural frequency spacing).

The distribution of natural frequencies of the MTMD can be made by varying either the stiffness or mass of each TMD. However, the manufacturing of a TMD with uniform stiffness and constant damping is simpler than that of one with varying stiffness and damping properties (the mass remains unchanged). Note that MTMDs with identical dynamic characteristics are equivalent to a single TMD in which the damping ratio and natural frequency of the equivalent single TMD are the same as those of the individual MTMD. However, the mass is the sum of all the MTMDs masses. The model considered above is the same as given by Xu and Igusa (1991).

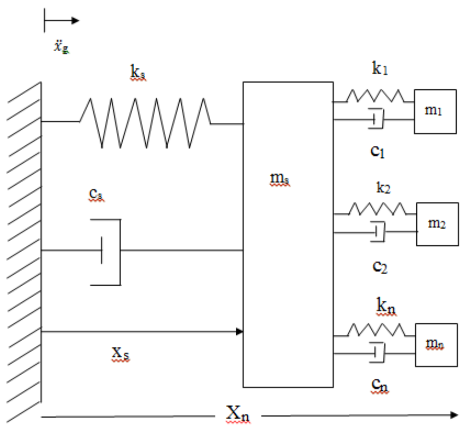

Fig. 4: A multiple tuned mass damper system

However, for the sake of completeness, the system parameters used for the present optimization study are briefly summarized below.

Let ω_T be the average frequency of all MTMDs ($\omega_T = \sum_{j=1}^{n}\omega_j/n$) where n is the total number of MTMDs. The natural frequency of the jth TMD is expressed as

$$\omega_j = \omega_T\left[1 + \left(j - \frac{n+1}{2}\right)\frac{\beta}{n-1}\right] \quad (4.1)$$

where the parameter β is the non-dimensional frequency spacing of the MTMD, defined as

$$\beta = \frac{\omega_n - \omega_1}{\omega_T} \quad (4.2)$$

If k_T and c_T are the constant stiffness and damping of each TMD respectively, then the mass and damping ratio of the jth TMD are expressed as

$$m_j = \frac{k_T}{\omega_j^2} \quad (4.3)$$

$$\xi_j = \frac{c_T}{2 m_j \omega_j} = \left(\frac{c_T}{2 k_T}\right)\omega_j \quad (4.4)$$

Note that the masses of the MTMDs in equation (4.3) follow an inverse quadratic relation to the natural frequency of the TMD. However, it has been shown that the masses should follow an elliptical relation with frequency under optimal condition. A more complex relation has also been proposed in reference.

The average damping ratio of the MTMD is expressed as

$$\xi_T = \sum_{j=1}^{n}\frac{\xi_j}{n} = \frac{\omega_T c_T}{2 k_T} \quad (4.5)$$

The ratio of the total MTMDs mass to the main system's mass is defined as the mass ratio.

$$\gamma = \frac{\sum_{j=1}^{n} m_j}{m_s} = \frac{m_T}{m_s} \quad (4.6)$$

where m_T is the total mass of the MTMD and m_s is the mass of the main system. The constant stiffness and damping of each TMD may be evaluated using,

$$k_T = \frac{\gamma m_s}{\left(\sum_{j=1}^{n}\frac{1}{\omega_j^2}\right)} \quad (4.7)$$

$$c_T = \frac{2\xi_T \gamma m_s}{\left(\omega_T \sum_{j=1}^{n}\frac{1}{\omega_j^2}\right)} \quad (4.8)$$

The ratio of the average frequency of the MTMD to the natural frequency of the main system is defined as the tuning frequency ratio.

$$f = \frac{\omega_T}{\omega_s} \quad (4.9)$$

The structural system considered in the study is quite complex, and it is very tedious to obtain the expression for the optimum parameters in closed form. As a result, the optimum parameters are obtained using a numerical searching procedure. For a given ξ_s, γ and n the parameters of the MTMD are varied such that the r.m.s. displacement of the main system attains the minimum value. The constraints applied on the values of parameters ξ_T, β and f for the optimization study are as follows.

$$0 \le \xi_T < 1, \quad 0 \le \beta < 2, \quad f > 0 \quad (4.17)$$

The above conditions satisfy the conditions that (1) The natural frequencies of the TMDs are positive real and (2) the TMDs are under-damped. The optimum parameters of the MTMD system are obtained for four values of the main system damping (ξ_s= 0, 2, 5, 10 %).

3.5. The effect of the number of MTMD on the optimum parameters:

In Fig. 7 is shown the variation of the optimum parameters ξ_T^{opt}, β^{opt},f^{opt} and ξ_{eq}^{opt}, versus the number of tuned mass dampers, n for the mass ratio γ=1% and ξ_s= 0, 2,5 and 10%. The optimum damping ratio,ξ_T^{opt}, decreases sharply as the number of TMDs increases. The optimum damping ratio for a single TMD is much higher than that for the MTMD system. Furthermore, the optimum damping is insensitive to changes in the main system damping. Warburton has shown that the optimum damping of the single TMD is not influenced by the damping of the main system, and the same is also confirmed for the MTMD system. The optimum frequency bandwidth, β^opt, of the MTMD system increases with the increase in both the number of TMDs as well as the damping of the main structure, as shown in the Fig. 7.

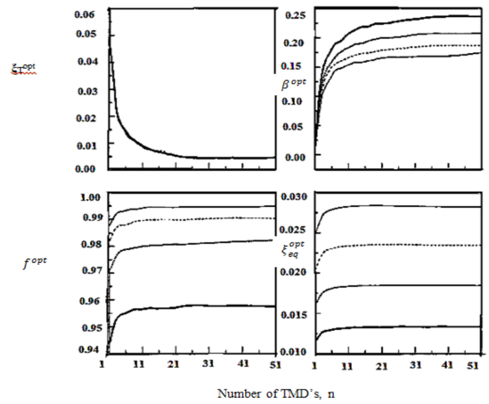

Fig. 5: Variation of the optimum parameters versus no. of tuned mass damper, for γ=1 % and ξ_s= 0, 2, 5, 10%.

Fig. 7 shows the variation of the optimum parameters versus no. of tuned mass damper, for γ=1 % and ξ_s= 0, 2, 5, 10%. The optimum tuning frequency ratio, f^opt, increases with an increase in the number of TMDs. However, it remains almost constant beyond a certain number of TMDs (in this case n>5). The optimum tuning frequency for single TMD is smaller than for the MTMD system. Also the optimum tuning frequency ratio decreases with an increase in the damping of the main structure. The equivalent damping, ξ_eq^opt, added to the main system at the optimum parameters shows a trend very similar to that of the optimum frequency bandwidth. There is an initial steep increase in the value of the equivalent damping. However, as the number of TMDs increases, the equivalent damping remains almost constant. The equivalent damping of the MTMD system is more than for a single TMD system. Thus, an optimum designed MTMD system is more effective than the optimum single TMD system. Furthermore, ξ_eq^opt, decreases as the main structure damping increases. This signifies that the effectiveness of the MTMD system decreases as the damping in the main structure increase.

3.6. The effect of the mass ratio on the optimum parameters

In this section, the variation of the optimum parameters ξ_T^opt, β^opt,f^opt and ξ_Teq^opt , versus the mass ratio, γ , is studied for different number of TMD, n=1,11 and 21.

The variation of the optimum frequency bandwidth of the MTMD system versus the mass ratio is shown in Fig. 8. As the mass ratio increases, the optimum frequency bandwidth also increases. The difference in the optimum frequency bandwidth between n=11 and n=21 increases mildly with the increase of mass ratio. It can also be seen from the figure that as the main structure damping increases, the optimum frequency bandwidth also increases for a given mass ratio and number of tuned mass dampers. Thus, the optimum frequency bandwidth of the MTMD system increases with an increase in both the mass ratio and the main system damping.

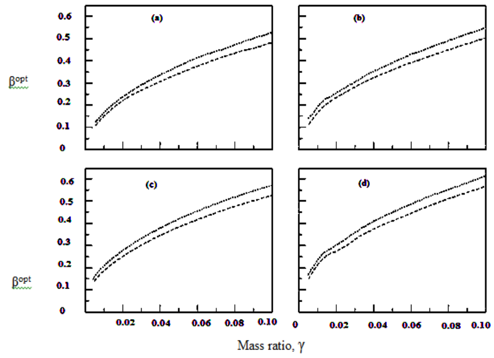

Fig. 6: Variation of the optimum frequency range of the MTMD versus mass ratio, for $f=f^{opt}$, $\beta=\beta^{opt}$ and $\xi_T=\xi_T^{opt}$ *(a)* ξ_s=0% *(b)* ξ_s=2% *(c)* ξ_s=5% *(d)* ξ_s=10% n= 1...... n=11......., n=21

In Fig. 8 variation of the optimum tuning frequency ratio, f^opt, is plotted versus the mass ratio the optimum tuning frequency ratio decreases with an increase in the mass ratio. The difference in the optimum tuning frequency ratio for n=11 and n=21 is not significant. For low values of the mass ratio, the optimum tuning ratio is the same for a single TMD and for the MTMD system. The optimum tuning ratio for a single TMD is much lower than that for the MTMD system for higher values of the mass ratio. At a given mass ratio, the optimum tuning frequency ratio increases with an increase in the number of TMDs and decreases with an increase in the main structure damping. Thus, the optimum tuning frequency ratio decreases with an increase in the mass ratio, being more pronounced for a single TMD as compared to the MTMD system.

In Fig. 9 is shown the variation of the equivalent damping of the TMD and MTMD added to the main system at the optimum parameters versus the mass ratio, γ. The optimum equivalent damping increases with an increase in the mass ratio. The equivalent damping of the optimum MTMD system is greater compared to that of the optimum single TMD. This indicates that an optimally designed MTMD system is more effective than a single TMD. The optimum equivalent damping added by the MTMD system and a single TMD is a maximum for the undamped system and decreases with an increase in the main system damping. Thus, the effectiveness of the MTMD system and the single TMD increases with an increase in the mass ratio. However, it is reduced for higher damping in the main system.

4. Results and conclusion

The stochastic response of a structure with the MTMD system subjected to base excitation is investigated. The base excitation is modeled as a stationary white noise random process.

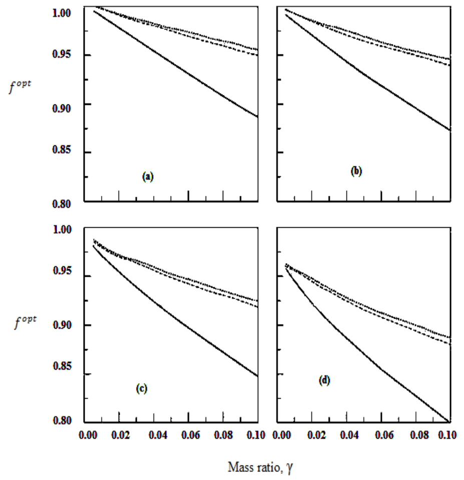

Fig. 7: Variation of the optimum tuning frequency ratio versus mass ratio for, f $=f^{opt}$, $\beta=\beta^{opt}$ and $\xi_T=\xi_T^{opt}$ *(a)* ξ_s=0% *(b)* ξ_s=2% *(c)* ξ_s=5% *(d)* ξ_s=10% n= 1...... n=11......., n=21.

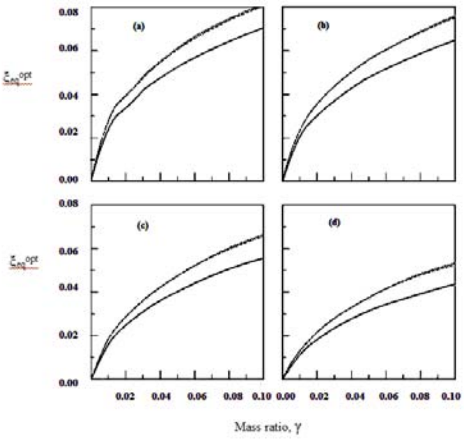

Fig. 8: Variation of the optimum equivalent damping ratio versus mass ratio, for $f=f^{opt}$, $\beta=\beta^{opt}$ and $\xi_T=\xi_T^{opt}$ *(a)* ξ_s=0% *(b)* ξ_s=2% *(c)* ξ_s=5% *(d)* ξ_s=10%.

The optimum parameters of the MTMD system are obtained for minimum r.m.s displacement of the main structure .The parameters of the MTMD system the damping ratio tuning frequency and frequency spacing are obtained for different numbers of TMDs and for different values of the mass ratio and the damping of the main structure In addition the optimum parameters of the MTMD system are compared with those corresponding to the single TMD system from the present study the following conclusions may be drawn.

1. For the same mass ratio the optimum designed MTMD system is found to be more effective than the optimum single TMD system.

2. The damping in the main system does not influence the optimum damping ratio of both the single TMD and the MTMD system.

3. The optimum frequency bandwidth of the MTMD system increases with an increase in both the mass ratio and the damping of the main system.

4. The effectiveness of the MTMD and the single TMD system is reduced for higher damping in the main system.

5. For the MTMD system the optimum damping ratio decreases whereas the frequency bandwidth increases mildly with an increase in the number of TMDs

6. The number of TMDs does not have much influence on the optimum tuning frequency and the corresponding effectiveness of the MTMD system.

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