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Optimizing system information by its Dimension

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Abstract: In order to analyze any business system its nature linear or nonlinear plays an important role. In case of nonlinear again the questions arises that how many factors are involve in the evolution of output value. In this paper an attempt is being made to get close to the answers of some of these and other issues. For this purpose an economic indicator data is used. In existing techniques, some linear regression methods and approximations are so far being successfully applied. Whereas using nonlinear dynamic theory, now it is possible for one to look more in-depth a system. Here a measure of nonlinear theory namely fractal dimension is calculated for a time series data of an economic indicator. This measure not only helps us to understand the nature (linear or nonlinear) of the system but also gives important information regarding system's sensitivity on initial conditions. The significance of duly weighted decision in any business scenario is priceless. This method has appeal of simplicity.

Key words: *Fractal dimension; Attractor; Phase space; Embedding dimension*

1. Introduction

The Efficient Market Hypothesis (EMH) basically says that current prices reflect all known information. This implies that there is a little or no correlation between returns; price changes occur in a random fashion, in creation to new information. This assumption – which has never been conclusively proven- is the bedrock upon which standard statistical analysis of the market has been built. The law of large numbers, for example, applies only if price changes are independent (i.e., "efficient"). And it is the law of large number that validates statistical calculus and other linear models.

Non-linear processes are the focus of research in many scientific areas. Although the beginnings of these new techniques have been in the natural sciences of physics and chemistry, they are now being applied to finance and economics. Interest in these techniques is based on the assumption that highly complex behavior that appears to be random is actually generated by an underlying nonlinear process. Typically, standard statistical tests, such as spectral analysis and autocorrelation function, are used to test for randomness and may fail to detect this hidden order (Thomas, 1995).

Empirical evidence of this phenomenon has been shown by various researchers. Brock and Sayers (1988) found nonlinearity in the U.S. labor market and investment. Barnett and Chen (1988) discovered low dimensionality in some U.S. monetary measures. Frank et al. (1988) indicated nonlinearities were present in Japan's quarterly real GDP. Strong non-linear dependence was also found in daily price changes of five foreign-exchange rates by Hsieh (1989) and in financial and agriculture futures by Blank (1990). Scheinkman and LeBaron (1989) found that a significant part of the variation in weekly Center for Research in Security Prices (CRSP) stock index returns is due to nonlinearities instead of randomness. Other studies in this area include Frank and Stengos (1988), Hinich and Patterosn (1985), and Savit (1988, 1989).

Linear model will prove successful only to the extent that the system being analyzed is itself linear. If the system is not linear, the models will work, at best only under ideal conditions and over short time periods. Thus the application of linear models to the market is questionable, in view of recent research suggesting that the capital markets, and the economy as a whole, may be governed in part by non-linear dynamics. When nonlinear dynamics are involved, a deterministic system can generate random-looking results that nevertheless exhibit persistent trends, cycles (both periodic and non-periodic), and long-term correlations.

Solution of dissipative dynamical systems cannot fill a volume of the phase-space; dissipation is synonymous with a contraction of volume elements under the action of equations of motion. Instead, trajectories are confined to lower dimensional subsets, which have measure zero in phase space. These subsets can be extremely complicated, and frequently they posses a fractal structure, which means that they are in a nontrivial way self-similar. Generalized dimensions are one class of quantities to characterize this fractality. The *Hausdorff dimension* is, from the mathematical point of view, the most natural concept to characterize fractal sets (Eckmann, 1985), whereas the *information*

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dimension takes into account the relative visitation frequencies and is therefore more attractive for physical systems. Finally, for the characterization of measured data, other similar concepts, like the *correlation dimension*, are more useful. One general remark is highly relevant in order to understand the limitations of any numerical approach: dimensions characterize a set or an invariant measure whose support is the set, whereas any data set contains only finite number of points representing the set to the measure. By definition, the dimension of a finite set of points is zero. When we determine the dimension of an attractor numerically, we extrapolate from finite length scales, where the statistics we apply is insensitive to the fitness of the number of data, to the infinitesimal scales, where the concept of dimension is defined. Dimensions are invariant under smooth transformations and thus again computable in time delay embedding space. An attractor is a state that defines equilibrium for a specific system. Equilibrium does not necessarily mean a "static" state, as econometric models define the term.

In this study correlation dimension is calculated for the daily, Karachi Inter Bank Offered Rate (KIBOR). This measure is used as a benchmark for banks to adjust their corporate loan settlements. It is not applicable for consumer landing. It is floating rate and operates on an open market basis. Central Bank monitors its fluctuation, and maintains it in stable range by using necessary financial instruments. These rates are offered from daily basis to a term of 3-years period. The data covers a period from September 2001 to February 2007, on daily basis (approx. 1641 data points).

2. Methodology

Constructing the phase-space of the time series under investigation, gives us important information of underlying attractor's geometry. Such a reconstruction approach uses the concept of embedding a single- variable series in a multi-dimensional phase-space. According to this approach, for a scalar time series, such as KIBOR (Karachi Inter-Bank Offer Rates), (Fig. 1), series, X_i, where i = 1, 2,….,N, the multi-dimensional phase-space can be reconstructed, using the method of delays,

$$Y_j = (X_j, X_{j+\lambda}, X_{j+2\lambda}, \ldots, X_{j+(m-1)\lambda}),$$

where $j = 1, 2, \ldots, N-(m-1)\lambda/\Delta t$; m is the dimension of the vector Y_j, called as the embedding dimension; and λ is a delay time taken to be some suitable multiple of the sampling time Δt. An appropriate delay time, λ, is essential for the phase-space reconstruction, since only an optimum λ gives the best separation of neighboring trajectories within the minimum embedding phase-space.

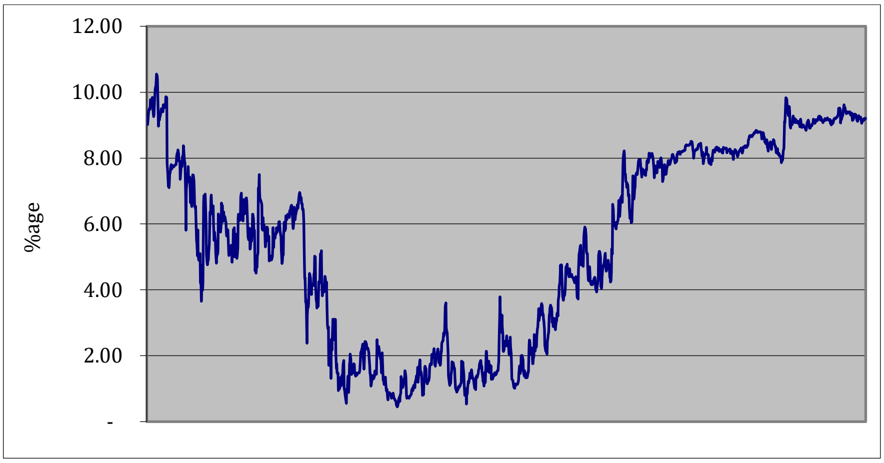

Fig. 1: KIBOR (Sep.'01 – Feb. '07)

The autocorrelation function has been the most widely used tool to determine λ. For instance, Holzfuss and Mayer-Kress (1986) suggest using value of delay time at which the autocorrelation function first crosses the zero line. Other approaches suggest the value of lag time at which the autocorrelation function attains a certain value, for instance, 0.5 and 0.1(Tsonis and Elsner, 1988).

In Fig. 2, we plot the ACF and PACF of the KIBOR series. It clearly indicates the most probable delay for the phase-space construction. After lag 1 the series PACF goes to zero and stays there, there is no sinusoidal damping.

Another way to get the system delay (λ) is by plotting one variable with different lags. Here lag 1(3a), 2 (3b), 7 (3c), and 100 (3d) are assumed. From the Fig. 3a, shows the most stable phase-space amongst the four. So, we can say that scalar time series has delay of one-value in between (i.e., λ = 1). This reconstructed phase-space has all the characteristics of the real phase space, provided the lag time and embedding dimension are properly specified (Peters, 1991). Both of these different techniques suggest the same time-delay value for the offered rates.

Sample: 1 1641
Included observations: 1641

Autocorrelation	Partial Correlation	Lag	AC	PAC
********	********	1	0.995	0.995
********		2	0.990	-0.043
********		3	0.986	0.018
********		4	0.981	-0.022
********		5	0.976	0.000
*******		6	0.971	0.046
*******		7	0.967	0.027
*******	*	8	0.963	0.092
*******	*	9	0.961	0.071
*******		10	0.959	0.053
*******		11	0.957	0.018
*******		12	0.955	0.004
*******		13	0.953	-0.023
*******		14	0.950	-0.033
*******		15	0.948	0.031
*******		16	0.945	0.017
*******		17	0.943	0.047
*******		18	0.941	0.007
*******		19	0.939	0.011
*******		20	0.936	0.001
*******		21	0.934	-0.039
*******		22	0.932	0.008
*******		23	0.930	0.003
*******		24	0.927	-0.029
*******		25	0.925	0.028
*******		26	0.922	0.000
*******		27	0.919	-0.025
*******		28	0.917	0.040
*******		29	0.915	-0.011
*******		30	0.912	-0.019
*******		31	0.910	0.019
*******		32	0.908	0.007
*******		33	0.906	0.001
*******		34	0.904	0.010
*******		35	0.901	-0.023
*******		36	0.899	-0.030

Fig. 2: AFC and PACF for KIBOR

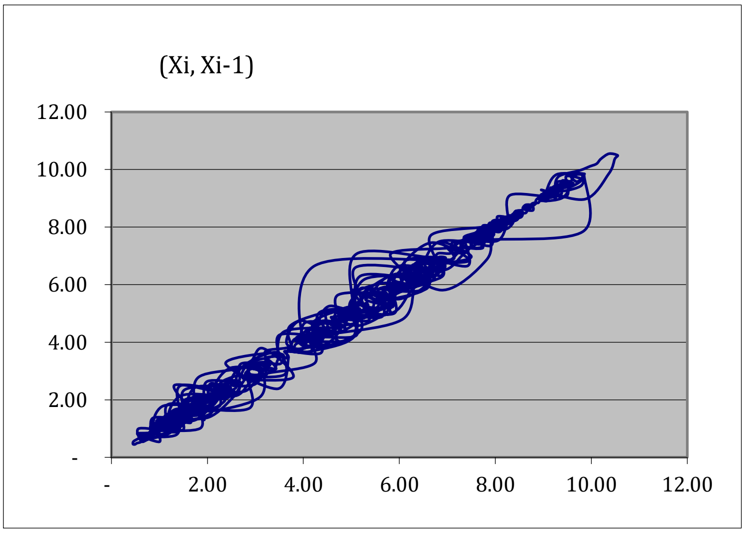

Fig. 3a: Lag 1 phase-space

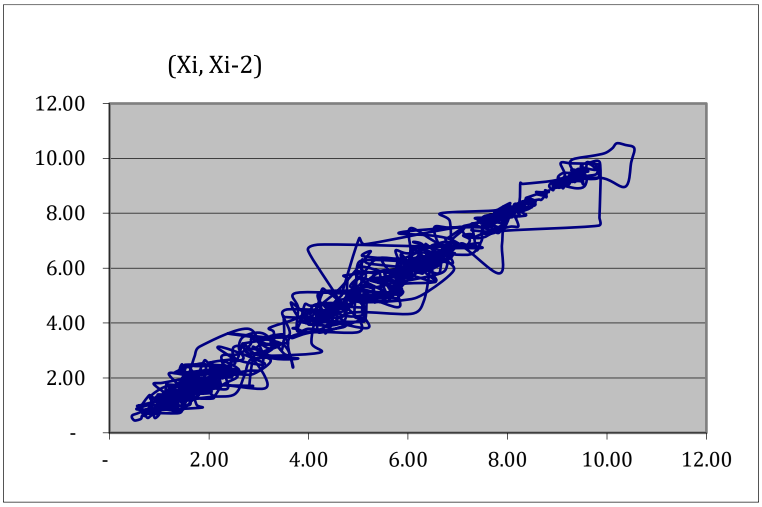

Fig. 3b: Lag 2 phase-space

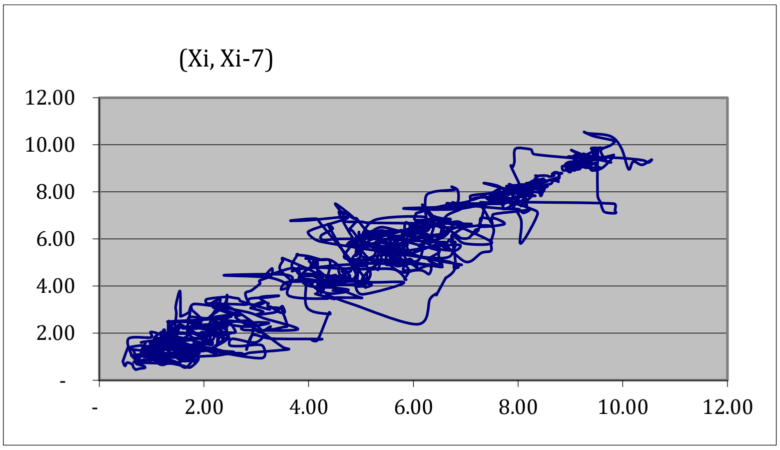

Figure 3c: Lag 7 phase-space

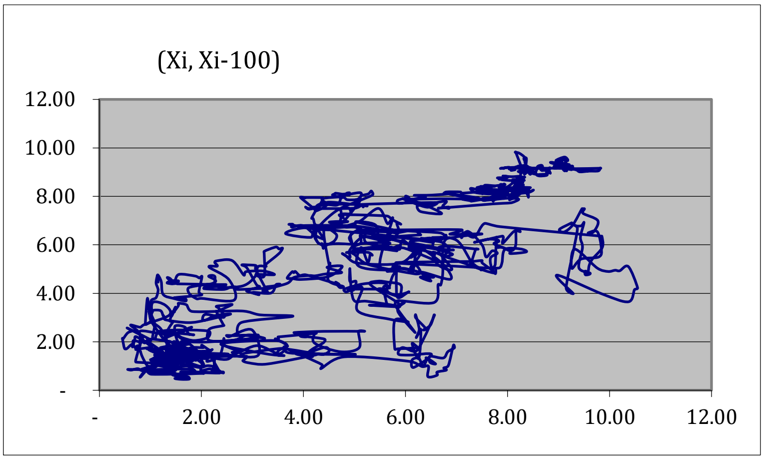

Fig. 3d: Lag 100 phase-space

The motivation for the fractal dimension of a phase-space is the information about the underlying attractor. More precisely, the next higher integer above the fractal dimension is the minimum number of variables we need to model the dynamics of the system. This gives us a lower bound on the number of degree of freedom in the whole system. It does not tell us what these variables are, but it can tell us something about the system's complexity. A low dimensional attractor of, say, three or four would suggest that the problem is solvable.

A pure random process, such as white noise, fills whatever space it is plotted in. In two dimensions, it fills the plane. In three dimensions, it fills the three-dimensional space, and so on for higher dimensions. In fact, white noise assumes the dimension of whatever space you place it in because its elements are uncorrelated and independent. The fractal dimension measures how an attractor fills its space. For chaotic attractor, the dimension is fractional; that is it is not an integer. Because chaotic attractor is deterministic, not every point in its phase space is equally likely, as it is with white noise.

Grassberger and Procaccia (1983) estimate the fractal dimension as the correlation dimension, *D*. *D* measures how densely the attractor fills its phase space by finding the probability that any one point will be a certain number distance, *R*, from another point. The correlation integral, $C_m(R)$, is the number of pairs of points in an *m-dimensional* phase space whose distance is less than *R*.

For a chaotic attractor, *C*m increases at a rate of R^D. this gives the following relation:

$C_m = R^D$
Or

$$\log(C_m) = D.\log(R) + \text{constant.} \quad (1)$$

By calculating the correlation integral, C_m, for various embedding dimensions, m, we can estimate *D* as the slope of a log/log plot of C_m and *R*. Grassberger and Procaccia have shown that, as *m* is increased, *D* will eventually converge to its true value i.e.,

$$D \approx \frac{Log(C_m)}{Log(R)} \quad (2)$$

The slope (*D*) is generally estimated by least-squares fit of a straight line over a certain range of *R*, called the scaling region.

Today's dynamicists (people who mathematically model the system dynamics) also use this measure (correlation dimension) as a significant signature of presence of chaos in particular (Natural and man-made system/phenomena) setups. Like if the correlation exponent attains saturation with an increase in the embedding dimension, (Islam and Sivakumar, 2002) then the system is generally considered to exhibit chaotic dynamics. The saturation value of the correlation exponent is defined as the correlation dimension of the attractor. The nearest integer above the saturation value provides the minimum or optimum embedding dimension (m_{opt}) for reconstructing the phase-space or the number of variable necessary to model the dynamics of the system.(Grassberger, 1986) On the other hand, if the correlation exponent increases without bound with increase in the embedding dimension, the system under investigation is generally considered stochastic. For stochastic process finite or low correlation dimension can also be observed (Osborne and Provenzale, 1989).

Sensitive Dependence of Attractor on Initial Conditions: Chaotic attractors are characterized by sensitive dependence on initial conditions. An error in measuring initial conditions will grow exponentially, so that a small error could dramatically affect forecasting ability. The further out in time we look, the less certain we are about the validity of our forecasts.

3. Results and Discussion

Here, the software package TISEAN is used for the calculation of required measures. Fig. 4 represents the correlation sum $C_m(R)$ at radius (R), for embedding dimensions 1 to 10. The correlation sum attains a value beyond which it seized to increase any further. When the slope at different embedding dimensions is plotted against correlation dimension the value attains the saturation approximately at 3.20 (Fig. 5). Further increase in embedding dimension does not increase the correlation dimension.

We have seen that the KIBOR has an underlying low dimensional attractor. The fractal dimension of this attractor is approximately 3.20. The attractor "chaotic", since the correlation dimension is fractional. And in order to create a reduced-noise-model we require at least four contributing variables for this purpose. All this indicates that there is an underlying, non-linear mechanism to the KIBOR. The exact form of this mechanism is unknown at this time. A likely candidate, however, is a mixed feedback system coupling the various forms of lending requirements of the banks for their liquidity to fulfill there future corporate obligations. Another possibility is the balance between the near future supply-demand scenario of money at prevailing interest rate.

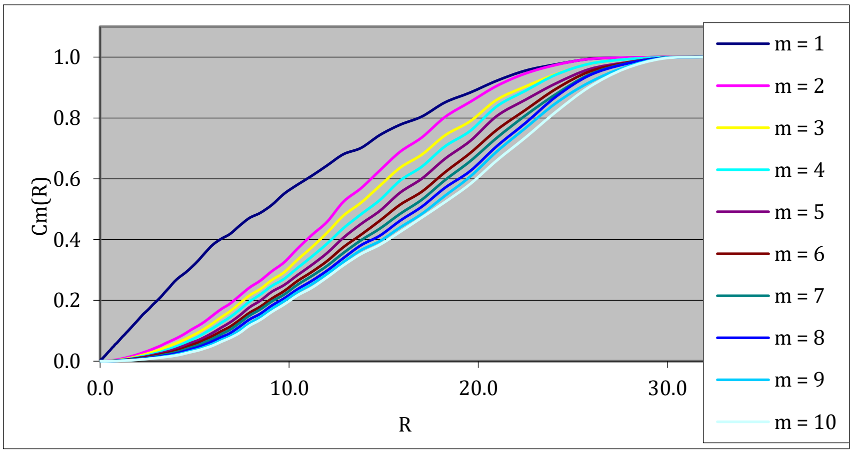

Fig. 4: Slopes of correlation sums

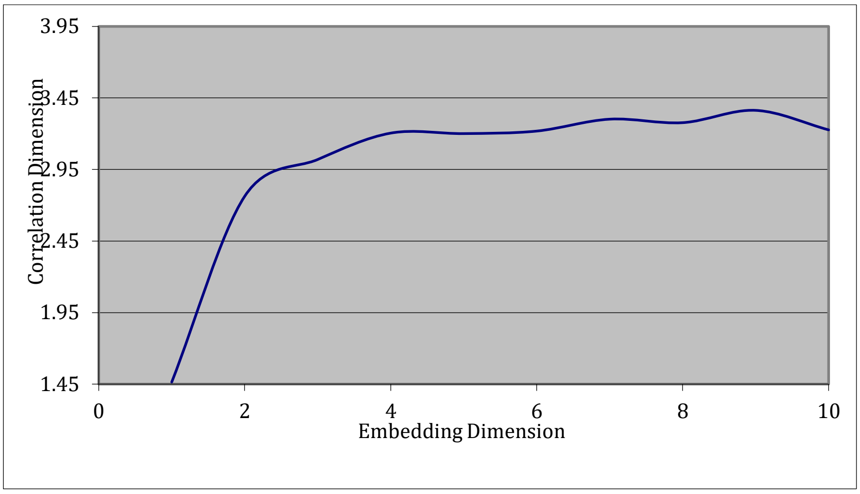

Fig. 5: Correlation dimension

On a broader scale, we can see why standard econometric methods have failed in the past. Attempts to "whiten" data in order to make it suitable for linear-based forecasting techniques cover up the conclusions nonlinear dynamics gives us. Surprisingly, these conclusions tie in with experience:

a. A small change in an indicator can have a major impact on the future.
b. The further out in time we go, the less reliable our forecasts are.

The second point is particularly important. It means that, as with the weather, accurate, long-range economic and market forecasting is not feasible from a practical standpoint. Even if were able to determine the equation of motion underlying the KIBOR, we would still not be able to forecast beyond a short time frame because we are never able to measure current conditions with significant accuracy. The future in quantitative economic theory should thus revolve around estimating the current probability distribution, which is not normal, so that we can better analyze the risk. Once that is done, viable economic and investment decisions can be made.

4. Summary and outlook

This attempt suggests that the KIBOR is stationary (Fig. 1). There are no seasonal and cyclic periods (or cycles). Now we also know that it has an underlying chaotic attractor, which may involve the other related economic (or money lending) indicators. Further we can affirm this finding by calculating other measure of attractor being chaotic. In this regard we can look into the Lyapunov Exponent. It measures the rate of diverging/ converging of trajectories, which starts from some initial values, as the system evolves. A positive value of Lyapunov Exponent is also a strong signature of attractor being chaotic.

One can look into develop a model that explains the nonlinear mechanism that generates the attractor. For this purpose nonlinear modeling techniques are most efficient. In this pursuit Mutual Information (MI) can be used to calculate the delay for the strange attractor. Since this method works better than autocorrelation function in some situations, and equally applicable for linear systems.

Finally, the efficient Market Hypothesis confirms to none of these observations. Neither does that standard econometrics, which is based on static equilibrium assumptions. If the social sciences are to grow, consideration should be given to defining equilibrium in a more dynamic fashion. The next phase involves models that explain these dynamics and statistical techniques for estimating risk. There is still much work to be done.

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